

Math

Module 2

(27 questions)

QUESTION 1

Choice D is correct. Since 1 yard is equal to 3 feet, 64 yards is equal to $64 \text{ yards} \left(\frac{3 \text{ feet}}{1 \text{ yard}} \right)$, or 192 feet. It follows that 64 yards per second is equivalent to 192 feet per second. Therefore, the object's speed is 192 feet per second.

Choice A is incorrect. A speed of 61 feet per second is equivalent to $\frac{61}{3}$, not 64, yards per second. **Choice B** is incorrect. A speed of 67 feet per second is equivalent to $\frac{67}{3}$, not 64, yards per second. **Choice C** is incorrect. A speed of 94 feet per second is equivalent to $\frac{94}{3}$, not 64, yards per second.

QUESTION 2

Choice A is correct. The line of best fit shown has a positive slope and intersects the y -axis at a positive y -value. The graph of an equation of the form $y = mx + b$, where m and b are constants, has a slope of m and intersects the y -axis at a y -value of b . Of the given choices, only $y = x + 3.4$ represents a line that has a positive slope, 1, and intersects the y -axis at a positive y -value, 3.4.

Choice B is incorrect. This equation represents a line that intersects the y -axis at a negative y -value, not a positive y -value. **Choice C** is incorrect. This equation represents a line that has a negative slope, not a positive slope. **Choice D** is incorrect. This equation represents a line that has a negative slope, not a positive slope, and intersects the y -axis at a negative y -value, not a positive y -value.

QUESTION 3

Choice D is correct. It's given that the graph shows the linear relationship between x and y . The given graph passes through the points $(0, -5)$, $(1, -3)$, and $(2, -1)$. It follows that when $x = 0$, the corresponding value of y is -5 , when $x = 1$, the corresponding value of y is -3 , and when $x = 2$, the corresponding value of y is -1 . Of the given choices, only the table in choice D gives these three values of x and their corresponding values of y for the relationship shown in the graph.

Choice A is incorrect. This table represents a relationship between x and y such that the graph passes through the points $(0, 0)$, $(1, -7)$, and $(2, -9)$. *Choice B* is incorrect. This table represents a relationship between x and y such that the graph passes through the points $(0, 0)$, $(1, -3)$, and $(2, -1)$. *Choice C* is incorrect. This table represents a linear relationship between x and y such that the graph passes through the points $(0, -5)$, $(1, -7)$, and $(2, -9)$.

QUESTION 4

Choice D is correct. The perimeter of a figure is equal to the sum of the measurements of the sides of the figure. It's given that the rectangle has a length of 4 inches and a width of 9 inches. Since a rectangle has 4 sides, of which opposite sides are parallel and equal, it follows that the rectangle has two sides with a length of 4 inches and two sides with a width of 9 inches. Therefore, the perimeter of this rectangle is $4 + 4 + 9 + 9$, or 26 inches.

Choice A is incorrect. This is the sum, in inches, of the length and the width of the rectangle. *Choice B* is incorrect. This is the sum, in inches, of the two lengths and the width of the rectangle. *Choice C* is incorrect. This is the sum, in inches, of the length and the two widths of the rectangle.

QUESTION 5

Choice A is correct. Dividing each side of the given equation by 7 yields

$$m = \frac{2(n+p)}{7}.$$

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This equation is equivalent to $7 + m = 2(n + p)$, not $7m = 2(n + p)$. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 6

The correct answer is 79. The median of a data set with an odd number of values is the middle value of the set when the values are ordered from least to greatest. Because the given data set consists of nine values that are ordered from least to greatest, the median is the fifth value in the data set. Therefore, the median of the data shown is 79.

QUESTION 7

The correct answer is 2. Substituting 8 for $f(x)$ in the given equation yields $8 = 4x$. Dividing the left- and right-hand sides of this equation by 4 yields $x = 2$. Therefore, the value of x is 2 when $f(x) = 8$.

QUESTION 8

Choice D is correct. The proportion of the paper clips that are size large can be written as $\frac{234,000}{300,000}$, or 0.78. Therefore, the percentage of the paper clips that are size large is $0.78(100)$, or 78%.

Choice A is incorrect. This is the percentage of the paper clips that are not size large. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice C* is incorrect and may result from conceptual or calculation errors.

QUESTION 9

Choice D is correct. It's given that the function $f(x) = 8x + 4$ gives the estimated height, in feet, of a willow tree x years after its height was first measured. For a function defined by an equation of the form $f(x) = mx + b$, where m and b are constants, b represents the value of $f(x)$ when $x = 0$. It follows that in the given function, 4 represents the value of $f(x)$ when $x = 0$. Therefore, the best interpretation of 4 in this context is that the estimated height of the tree was 4 feet when it was first measured.

Choice A is incorrect and may result from conceptual errors. *Choice B* is incorrect and may result from conceptual errors. *Choice C* is incorrect and may result from conceptual errors.

QUESTION 10

Choice B is correct. Since the point (x, y) is an intersection point of the graphs of the given equations in the xy -plane, the pair (x, y) should satisfy both equations, and thus is a solution of the given system. According to the first equation, $y = 76$. Substituting 76 in place of y in the second equation yields $x^2 - 5 = 76$. Adding 5 to both sides of this equation yields $x^2 = 81$. Taking the square root of both sides of this equation yields two solutions: $x = 9$ and $x = -9$. Of these two solutions, only -9 is given as a choice.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the value of coordinate y , rather than x , of the intersection point (x, y) .

QUESTION 11

Choice A is correct. It's given that each side of equilateral triangle S is multiplied by a scale factor of k to create equilateral triangle T. Since the length of each side of triangle T is greater than the length of each side of triangle S, the scale factor of k must be greater than 1. Of the given choices, only $\frac{29}{28}$ is greater than 1.

Choice B is incorrect. If each side of equilateral triangle S is multiplied by a scale factor of 1, the length of each side of triangle T would be equal to the length of each side of triangle S. *Choice C* is incorrect. If each side of equilateral triangle S is multiplied by a scale factor of $\frac{28}{29}$, the length of each side of triangle T would be less than the length of each side of triangle S. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 12

Choice C is correct. If the two sides of a linear equation are equivalent, then the equation is true for any value. If an equation is true for any value, it has infinitely many solutions. Since the two sides of the given linear equation $66x = 66x$ are equivalent, the given equation has infinitely many solutions.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 13

The correct answer is 41. The number of cupcakes Vivian bought can be found by first finding the amount Vivian spent on cupcakes. The amount Vivian spent on cupcakes can be found by subtracting the amount Vivian spent on party hats from the total amount Vivian spent. The amount Vivian spent on party hats can be found by multiplying the cost per package of party hats by the number of packages of party hats, which yields $\$3 \cdot 10$, or $\$30$. Subtracting the amount Vivian spent on party hats, $\$30$, from the total amount Vivian spent, $\$71$, yields $\$71 - \$30$, or $\$41$. Since the amount Vivian spent on cupcakes was $\$41$ and each cupcake cost $\$1$, it follows that Vivian bought 41 cupcakes.

QUESTION 14

The correct answer is 11,875. It's given that the exponential function g is defined by $g(x) = 19 \cdot a^x$, where a is a positive constant, and $g(3) = 2,375$. It follows that when $x = 3$, $g(x) = 2,375$. Substituting 3 for x and 2,375 for $g(x)$ in the given equation yields $2,375 = 19 \cdot a^3$. Dividing each side of this equation by 19 yields $125 = a^3$. Taking the cube root of both sides of this equation gives $a = 5$.

Substituting 4 for x and 5 for a in the equation $g(x) = 19 \cdot a^x$ yields $g(4) = 19 \cdot 5^4$, or $g(4) = 11,875$. Therefore, the value of $g(4)$ is 11,875.

QUESTION 15

Choice B is correct. The sine of any acute angle is equal to the cosine of its complement. It's given that in right triangle RST , the sum of the measures of angle R and angle S is 90 degrees. Therefore, angle R and angle S are complementary, and the value of $\sin R$ is equal to the value of $\cos S$. It's given that the value of $\sin R$ is $\frac{\sqrt{15}}{4}$, so the value of $\cos S$ is also $\frac{\sqrt{15}}{4}$.

Choice A is incorrect. This is the value of $\tan S$. *Choice C* is incorrect. This is the value of $\frac{1}{\cos S}$. *Choice D* is incorrect. This is the value of $\frac{1}{\tan S}$.

QUESTION 16

Choice B is correct. The graph shown is a line passing through the points $(0, 40)$ and $(60, 0)$. Since the relationship between x and y is linear, if two points on the graph make a linear equation true, then the equation represents the relationship. Substituting 0 for x and 40 for y in the equation in choice B,

$8x + 12y = 480$, yields $8(0) + 12(40) = 480$, or $480 = 480$, which is true. Substituting 60 for x and 0 for y in the equation $8x + 12y = 480$ yields $8(60) + 12(0) = 480$, or $480 = 480$, which is true. Therefore, the equation $8x + 12y = 480$ represents the relationship between x and y .

Choice A is incorrect. The point $(0, 40)$ is not on the graph of this equation, since $40 = 8(0) + 12$, or $40 = 12$, is not true. *Choice C* is incorrect. The point $(0, 40)$ is not on the graph of this equation, since $40 = 12(0) + 8$, or $40 = 8$, is not true. *Choice D* is incorrect. The point $(0, 40)$ is not on the graph of this equation, since $12(0) + 8(40) = 480$, or $320 = 480$, is not true.

QUESTION 17

Choice B is correct. The given expression has a common factor of 2 in the denominator, so the expression can be rewritten as $\frac{8x(x-7)-3(x-7)}{2(x-7)}$. The three terms in this expression have a common factor of $(x-7)$. Since it's given that $x > 7$, x can't be equal to 7, which means $(x-7)$ can't be equal to 0. Therefore, each term in the expression, $\frac{8x(x-7)-3(x-7)}{2(x-7)}$, can be divided by $(x-7)$, which gives $\frac{8x-3}{2}$.

Choice A is incorrect and may result from conceptual or calculation errors.
Choice C is incorrect and may result from conceptual or calculation errors.
Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 18

Choice A is correct. The y -intercept of the graph of $y = f(x)$ in the xy -plane occurs at the point on the graph where $x = 0$. In other words, when $x = 0$, the corresponding value of $f(x)$ is the y -coordinate of the y -intercept. Substituting 0 for x in the given equation yields $f(0) = (-8)(2)^0 + 22$, which is equivalent to $f(0) = (-8)(1) + 22$, or $f(0) = 14$. Thus, when $x = 0$, the corresponding value of $f(x)$ is 14. Therefore, the y -intercept of the graph of $y = f(x)$ in the xy -plane is $(0, 14)$.

Choice B is incorrect and may result from conceptual or calculation errors.
Choice C is incorrect and may result from conceptual or calculation errors.
Choice D is incorrect. This could be the y -intercept for $f(x) = (-8)(2)^x$, not $f(x) = (-8)(2)^x + 22$.

QUESTION 19

Choice C is correct. It's given that the equation $3x + 5y = 32$ represents the situation where Keenan filled x small jars and y large jars with all the vegetable broth he made, which was 32 cups. Therefore, $3x$ represents the total number of cups of vegetable broth in the small jars and $5y$ represents the total number of cups of vegetable broth in the large jars.

Choice A is incorrect. The number of large jars Keenan filled is represented by y , not $5y$. *Choice B* is incorrect. The number of small jars Keenan filled is represented by x , not $5y$. *Choice D* is incorrect. The total number of cups of vegetable broth in the small jars is represented by $3x$, not $5y$.

QUESTION 20

The correct answer is 5. The standard form of an equation of a circle in the xy -plane is $(x-h)^2 + (y-k)^2 = r^2$, where h , k , and r are constants, the coordinates of the center of the circle are (h, k) , and the length of the radius of the circle is r . It's given that an equation of the circle is $(x-2)^2 + (y-9)^2 = r^2$.

Therefore, the center of this circle is $(2, 9)$. It's given that the endpoints of a diameter of the circle are $(2, 4)$ and $(2, 14)$. The length of the radius is the distance from the center of the circle to an endpoint of a diameter of the circle, which can be found using the distance formula, $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Substituting the center of the circle $(2, 9)$ and one endpoint of the diameter $(2, 4)$ in this formula gives a distance of $\sqrt{(2-2)^2 + (9-4)^2}$, or $\sqrt{0^2 + 5^2}$, which is equivalent to 5. Since the distance from the center of the circle to an endpoint of a diameter is 5, the value of r is 5.

QUESTION 21

The correct answer is $\frac{1}{4}$. For an equation in slope-intercept form $y = mx + b$, m represents the slope of the line in the xy -plane defined by this equation. It's given that line ℓ is defined by $3y + 12x = 5$. Subtracting $12x$ from both sides of this equation yields $3y = -12x + 5$. Dividing both sides of this equation by 3 yields $y = -\frac{12}{3}x + \frac{5}{3}$, or $y = -4x + \frac{5}{3}$. Thus, the slope of line ℓ in the xy -plane is -4 .

Since line n is perpendicular to line ℓ in the xy -plane, the slope of line n is the negative reciprocal of the slope of line ℓ . The negative reciprocal of -4 is $-\frac{1}{(-4)} = \frac{1}{4}$. Note that $1/4$ and $.25$ are examples of ways to enter a correct answer.

QUESTION 22

Choice D is correct. By the definition of absolute value, if $|-5x + 13| = 73$, then $-5x + 13 = 73$ or $-5x + 13 = -73$. Subtracting 13 from both sides of the equation $-5x + 13 = 73$ yields $-5x = 60$. Dividing both sides of this equation by -5 yields $x = -12$. Subtracting 13 from both sides of the equation $-5x + 13 = -73$ yields $-5x = -86$. Dividing both sides of this equation by -5 yields $x = \frac{86}{5}$. Therefore, the solutions to the given equation are -12 and $\frac{86}{5}$, and it follows that the sum of the solutions to the given equation is $-12 + \frac{86}{5}$, or $\frac{26}{5}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is a solution, not the sum of the solutions, to the given equation. **Choice C** is incorrect and may result from conceptual or calculation errors.

QUESTION 23

Choice C is correct. For the form of the function in choice C, $f(x) = 128(1.6)^{x-1}$, the value of $f(1)$ can be found as $128(1.6)^{1-1}$, which is equivalent to $128(1.6)^0$, or 128. Therefore, $k = 128$, which is shown in $f(x) = 128(1.6)^{x-1}$ as the coefficient.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 24

Choice C is correct. It's given that the equation $-9x^2 + 30x + c = 0$ has exactly one solution. A quadratic equation of the form $ax^2 + bx + c = 0$ has exactly one solution if and only if its discriminant, $-4ac + b^2$, is equal to zero. It follows that for the given equation, $a = -9$ and $b = 30$. Substituting -9 for a and 30 for b into $b^2 - 4ac$ yields $30^2 - 4(-9)(c)$, or $900 + 36c$. Since the discriminant must equal zero, $900 + 36c = 0$. Subtracting $36c$ from both sides of this equation yields $900 = -36c$. Dividing each side of this equation by -36 yields $-25 = c$. Therefore, the value of c is -25 .

Choice A is incorrect. If the value of c is 3 , this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution. **Choice B** is incorrect. If the value of c is 0 , this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution. **Choice D** is incorrect. If the value of c is -53 , this would yield a discriminant that is less than zero. Therefore, the given equation would have no real solutions, rather than exactly one solution.

QUESTION 25

Choice D is correct. Since each choice has a term of $3x^2$, which can be written as $(3x)(x)$, and each choice has a term of $14b$, which can be written as $(7)(2b)$, the expression that has a factor of $x + 2b$, where b is a positive integer constant, can be represented as $(3x + 7)(x + 2b)$. Using the distributive property of multiplication, this expression is equivalent to $3x(x + 2b) + 7(x + 2b)$, or $3x^2 + 6xb + 7x + 14b$. Combining the x -terms in this expression yields $3x^2 + (7 + 6b)x + 14b$. It follows that the coefficient of the x -term is equal to $7 + 6b$. Thus, from the given choices, $7 + 6b$ must be equal to 7 , 28 , 42 , or 49 . Therefore, $6b$ must be equal to 0 , 21 , 35 , or 42 , respectively, and b must be equal to $\frac{0}{6}$, $\frac{21}{6}$, $\frac{35}{6}$, or $\frac{42}{6}$, respectively. Of these four values of b , only $\frac{42}{6}$, or 7 , is a positive integer. It follows that $7 + 6b$ must be equal to 49 because this is the only choice for which the value of b is a positive integer constant. Therefore, the expression that has a factor of $x + 2b$ is $3x^2 + 49x + 14b$.

Choice A is incorrect. If this expression has a factor of $x + 2b$, then the value of b is 0 , which isn't positive. **Choice B** is incorrect. If this expression has a factor of $x + 2b$, then the value of b is $\frac{21}{6}$, which isn't an integer. **Choice C** is incorrect. If this expression has a factor of $x + 2b$, then the value of b is $\frac{35}{6}$, which isn't an integer.

QUESTION 26

Choice B is correct. The histograms shown have the same shape, but data set A contains values between 20 and 60 and data set B contains values between 10 and 50 . Thus, the mean of data set A is greater than the mean of data set B. Therefore, the smallest possible difference between the mean of data set A and the mean of data set B is the difference between the smallest possible mean of data set A and the greatest possible mean of data set B. In data set A, since there

are 3 integers in the interval greater than or equal to 20 but less than 30, 4 integers greater than or equal to 30 but less than 40, 7 integers greater than or equal to 40 but less than 50, and 9 integers greater than or equal to 50 but less than 60, the smallest possible mean for data set A is $\frac{(3 \cdot 20) + (4 \cdot 30) + (7 \cdot 40) + (9 \cdot 50)}{23}$. In data set B, since there are 3 integers greater than or equal to 10 but less than 20, 4 integers greater than or equal to 20 but less than 30, 7 integers greater than or equal to 30 but less than 40, and 9 integers greater than or equal to 40 but less than 50, the largest possible mean for data set B is $\frac{(3 \cdot 19) + (4 \cdot 29) + (7 \cdot 39) + (9 \cdot 49)}{23}$. Therefore, the smallest possible difference between the mean of data set A and the mean of data set B is $\frac{(3 \cdot 20) + (4 \cdot 30) + (7 \cdot 40) + (9 \cdot 50)}{23} - \frac{(3 \cdot 19) + (4 \cdot 29) + (7 \cdot 39) + (9 \cdot 49)}{23}$, which is equivalent to $\frac{(3 \cdot 20) - (3 \cdot 19) + (4 \cdot 30) - (4 \cdot 29) + (7 \cdot 40) - (7 \cdot 39) + (9 \cdot 50) - (9 \cdot 49)}{23}$. This expression can be rewritten as $\frac{3(20 - 19) + 4(30 - 29) + 7(40 - 39) + 9(50 - 49)}{23}$, or $\frac{23}{23}$, which is equal to 1. Therefore, the smallest possible difference between the mean of data set A and the mean of data set B is 1.

Choice A is incorrect. This is the smallest possible difference between the ranges, not the means, of the data sets. *Choice C* is incorrect. This is the difference between the greatest possible mean, not the smallest possible mean, of data set A and the greatest possible mean of data set B. *Choice D* is incorrect. This is the smallest possible difference between the sum of the values in data set A and the sum of the values in data set B, not the smallest possible difference between the means.

QUESTION 27

The correct answer is 104. An equilateral triangle is a triangle in which all three sides have the same length and all three angles have a measure of 60° . The height of the triangle, $k\sqrt{3}$, is the length of the altitude from one vertex. The altitude divides the equilateral triangle into two congruent 30-60-90 right triangles, where the altitude is the side across from the 60° angle in each 30-60-90 right triangle. Since the altitude has a length of $k\sqrt{3}$, it follows from the properties of 30-60-90 right triangles that the side across from each 30° angle has a length of k and each hypotenuse has a length of $2k$. In this case, the hypotenuse of each 30-60-90 right triangle is a side of the equilateral triangle; therefore, each side length of the equilateral triangle is $2k$. The perimeter of a triangle is the sum of the lengths of each side. It's given that the perimeter of the equilateral triangle is 624; therefore, $2k + 2k + 2k = 624$, or $6k = 624$. Dividing both sides of this equation by 6 yields $k = 104$.