

Math

Module 1 (27 questions)

QUESTION 1

Choice C is correct. For the given line graph, the percent of cars for sale at a used car lot on a given day is represented on the vertical axis. The percent of cars for sale is the smallest when the height of the line graph is the lowest. The lowest height of the line graph occurs for cars with a model year of 2014.

Choice A is incorrect and may result from conceptual errors. *Choice B* is incorrect and may result from conceptual errors. *Choice D* is incorrect and may result from conceptual errors.

QUESTION 2

Choice C is correct. It's given that 29 out of every 100 beads that the machine produces have a defect. It follows that if the machine produces k beads, then the number of beads that have a defect is $\frac{29}{100}k$, for some constant k . If a bead produced by the machine will be selected at random, the probability of selecting a bead that has a defect is given by the number of beads with a defect, $\frac{29}{100}k$, divided by the number of beads produced by the machine, k . Therefore, the probability of selecting a bead that has a defect is $\frac{\frac{29}{100}k}{k}$, or $\frac{29}{100}$.

Choice A is incorrect and may result from conceptual or computational errors. *Choice B* is incorrect and may result from conceptual or computational errors. *Choice D* is incorrect and may result from conceptual or computational errors.

QUESTION 3

Choice D is correct. It's given that line m is parallel to line n , and line t intersects both lines. It follows that line t is a transversal. When two lines are parallel and intersected by a transversal, exterior angles on the same side of the transversal

are supplementary. Thus, $x + 33 = 180$. Subtracting 33 from both sides of this equation yields $x = 147$. Therefore, the value of x is 147.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 4

Choice D is correct. The y -intercept of a graph in the xy -plane is the point at which the graph crosses the y -axis. The graph shown crosses the y -axis at the point $(0, 8)$. Therefore, the y -intercept of the graph shown is $(0, 8)$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 5

Choice C is correct. It's given that $f(x)$ is the total cost, in dollars, to lease a car from this dealership with a monthly payment of x dollars. Therefore, the total cost, in dollars, to lease the car when the monthly payment is \$400 is represented by the value of $f(x)$ when $x = 400$. Substituting 400 for x in the equation $f(x) = 36x + 1,000$ yields $f(400) = 36(400) + 1,000$, or $f(400) = 15,400$. Thus, when the monthly payment is \$400, the total cost to lease a car is \$15,400.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 6

The correct answer is 180. The perimeter of a polygon is equal to the sum of the lengths of the sides of the polygon. It's given that each side of the square has a length of 45. Since a square is a polygon with 4 sides, the perimeter of this square is $45 + 45 + 45 + 45$, or 180.

QUESTION 7

The correct answer is 5. Multiplying both sides of the given equation by $x + 6$ results in $55 = x(x + 6)$. Applying the distributive property of multiplication to the right-hand side of this equation results in $55 = x^2 + 6x$. Subtracting 55 from both sides of this equation results in $0 = x^2 + 6x - 55$. The right-hand side of this equation can be rewritten by factoring. The two values that multiply to -55 and add to 6 are 11 and -5 . It follows that the equation $0 = x^2 + 6x - 55$ can be rewritten as $0 = (x + 11)(x - 5)$. Setting each factor equal to 0 yields two equations: $x + 11 = 0$ and $x - 5 = 0$. Subtracting 11 from both sides of the equation $x + 11 = 0$ results in $x = -11$. Adding 5 to both sides of the equation $x - 5 = 0$ results in $x = 5$. Therefore, the positive solution to the given equation is 5.

QUESTION 8

Choice A is correct. If the object travels 108 centimeters at a speed of 12 centimeters per second, the time of travel can be determined by dividing the total distance by the speed. This results in $\frac{108 \text{ centimeters}}{12 \text{ centimeters/second}}$, which is 9 seconds.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 9

Choice B is correct. The mean of a data set is the sum of the values in the data set divided by the number of values in the data set. It follows that the mean of data set X is $\frac{5+9+9+13}{4}$, or 9, and the mean of data set Y is $\frac{5+9+9+13+27}{5}$, or 12.6. Since 9 is less than 12.6, the mean of data set X is less than the mean of data set Y.

Alternate approach: Data set Y consists of the 4 values in data set X and one additional value, 27. Since the additional value, 27, is larger than any value in data set X, the mean of data set X is less than the mean of data set Y.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 10

Choice A is correct. It's given that the rocket contained 467,000 kilograms (kg) of propellant before launch and had 362,105 kg remaining exactly 21 seconds after launch. Finding the difference between the amount, in kg, of propellant before launch and the remaining amount, in kg, of propellant after launch gives the amount, in kg, of propellant burned during the 21 seconds: $467,000 - 362,105 = 104,895$. Dividing the amount of propellant burned by the number of seconds yields $\frac{104,895}{21} = 4,995$. Thus, an average of 4,995 kg of propellant burned each second after launch.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from finding the amount of propellant burned, rather than the amount of propellant burned each second.

QUESTION 11

Choice B is correct. Multiplying both sides of the given equation by 4 yields $(4)(4x+2) = (4)(12)$, or $16x+8 = 48$. Therefore, the value of $16x+8$ is 48.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 12

Choice D is correct. It's given that the equation $h = -4.9t^2 + 7t + 9$ represents this situation, where h is the height, in meters, of the object t seconds after it is kicked. It follows that the height, in meters, from which the object was kicked is the value of h when $t = 0$. Substituting 0 for t in the equation $h = -4.9t^2 + 7t + 9$ yields $h = -4.9(0)^2 + 7(0) + 9$, or $h = 9$. Therefore, the object was kicked from a height of 9 meters.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 13

The correct answer is $\frac{25}{4}$. The given equation can be rewritten in the

form $f(x) = a(x - h)^2 + k$, where a , h , and k are constants. When $a > 0$,

h is the value of x for which $f(x)$ reaches its minimum. The given equation

can be rewritten as $f(x) = 4\left(x^2 - \frac{50}{4}x\right) + 126$, which is equivalent to

$f(x) = 4\left(x^2 - \frac{50}{4}x + \left(\frac{50}{8}\right)^2 - \left(\frac{50}{8}\right)^2\right) + 126$. This equation can be rewritten as

$f(x) = 4\left(\left(x - \frac{50}{8}\right)^2 - \left(\frac{50}{8}\right)^2\right) + 126$, or $f(x) = 4\left(x - \frac{50}{8}\right)^2 - 4\left(\frac{50}{8}\right)^2 + 126$, which is

equivalent to $f(x) = 4\left(x - \frac{25}{4}\right)^2 - \frac{121}{4}$. Therefore, $h = \frac{25}{4}$, so the value of x for which

$f(x)$ reaches its minimum is $\frac{25}{4}$. Note that 25/4 and 6.25 are examples of ways to

enter a correct answer.

QUESTION 14

The correct answer is 182. Let s represent the number of small candles the owner can purchase, and let ℓ represent the number of large candles the owner can purchase. It's given that the owner pays \$4.90 per candle to purchase small candles and \$11.60 per candle to purchase large candles. Therefore, the owner pays $4.90s$ dollars for s small candles and 11.60ℓ dollars for ℓ large candles, which means the owner pays a total of $4.90s + 11.60\ell$ dollars to purchase candles. It's given that the owner budgets \$2,200 to purchase candles.

Therefore, $4.90s + 11.60\ell \leq 2,200$. It's also given that the owner must purchase a minimum of 200 candles. Therefore, $s + \ell \geq 200$. The inequalities

$4.90s + 11.60\ell \leq 2,200$ and $s + \ell \geq 200$ can be combined into one compound inequality by rewriting the second inequality so that its left-hand side is equivalent to the left-hand side of the first inequality. Subtracting ℓ from both sides of the inequality $s + \ell \geq 200$ yields $s \geq 200 - \ell$. Multiplying both sides of this inequality by 4.90 yields $4.90s \geq 4.90(200 - \ell)$, or $4.90s \geq 980 - 4.90\ell$. Adding 11.60ℓ to both sides of this inequality yields $4.90s + 11.60\ell \geq 980 - 4.90\ell + 11.60\ell$, or $4.90s + 11.60\ell \geq 980 + 6.70\ell$. This inequality can be combined with the inequality $4.90s + 11.60\ell \leq 2,200$, which yields the compound inequality

$980 + 6.70\ell \leq 4.90s + 11.60\ell \leq 2,200$. It follows that $980 + 6.70\ell \leq 2,200$.

Subtracting 980 from both sides of this inequality yields $6.70\ell \leq 2,200$. Dividing both sides of this inequality by 6.70 yields approximately $\ell \leq 182.09$. Since the number of large candles the owner purchases must be a whole number, the maximum number of large candles the owner can purchase is the largest whole number less than 182.09, which is 182.

QUESTION 15

Choice D is correct. Since f is a linear function, it can be defined by an equation of the form $f(x) = ax + b$, where a and b are constants. It's given that $f(0) = 8$. Substituting 0 for x and 8 for $f(x)$ in the equation $f(x) = ax + b$ yields $8 = a(0) + b$, or $8 = b$. Substituting 8 for b in the equation $f(x) = ax + b$ yields $f(x) = ax + 8$. It's given that $f(1) = 12$. Substituting 1 for x and 12 for $f(x)$ in the equation $f(x) = ax + 8$ yields $12 = a(1) + 8$, or $12 = a + 8$. Subtracting 8 from both sides of this equation yields $a = 4$. Substituting 4 for a in the equation $f(x) = ax + 8$ yields $f(x) = 4x + 8$. Therefore, an equation that defines f is $f(x) = 4x + 8$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 16

Choice A is correct. The function f gives the area of the rectangle, in ft^2 , if its width is w ft. Since the value of $f(14)$ is the value of $f(w)$ if $w = 14$, it follows that $f(14) = 1,176$ means that $f(w)$ is 1,176 if $w = 14$. In the given context, this means that if the width of the rectangle is 14 ft, then the area of the rectangle is $1,176 \text{ ft}^2$.

Choice B is incorrect and may result from conceptual errors. *Choice C* is incorrect and may result from conceptual errors. *Choice D* is incorrect and may result from interpreting $f(w)$ as the width, in ft, of the rectangle if its area is $w \text{ ft}^2$, rather than as the area, in ft^2 , of the rectangle if its width is w ft.

QUESTION 17

Choice B is correct. Since $\overline{PR}$ and $\overline{QS}$ are diameters of the circle shown, $\overline{OS}$, $\overline{OR}$, $\overline{OP}$, and $\overline{OQ}$ are radii of the circle and are therefore congruent. Since $\angle SOP$ and $\angle ROQ$ are vertical angles, they are congruent. Therefore, arc PS and arc QR are formed by congruent radii and have the same angle measure, so they are congruent arcs. Similarly, $\angle SOR$ and $\angle POQ$ are vertical angles, so they are congruent. Therefore, arc SR and arc PQ are formed by congruent radii and have the same angle measure, so they are congruent arcs. Let x represent the length of arc SR . Since arc SR and arc PQ are congruent arcs, the length of arc PQ can also be represented by x . It's given that the length of arc PS is twice the length of arc PQ . Therefore, the length of arc PS can be represented by the expression $2x$. Since arc PS and arc QR are congruent arcs, the length of arc QR can also be represented by $2x$. This gives the expression $x + x + 2x + 2x$.

Since it's given that the circumference is 144π , the expression $x + x + 2x + 2x$ is equal to 144π . Thus $x + x + 2x + 2x = 144\pi$, or $6x = 144\pi$. Dividing both sides of this equation by 6 yields $x = 24\pi$. Therefore, the length of arc QR is $2(24\pi)$, or 48π .

Choice A is incorrect. This is the length of arc PQ , not arc QR . *Choice C* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 18

Choice C is correct. Let x represent the number of children in a whale-watching tour group. Let y represent the number of adults in this group. Because it's given that 21 people are in a group and the group consists of adults and children, it must be true that $x + y = 21$. Since the company's revenue is 60 dollars per child, the total revenue from x children in this group was $60x$ dollars. Since the company's revenue is 80 dollars per adult, the total revenue from y adults in this group was $80y$ dollars. Because it's given that the total revenue for this group was 1,440 dollars, it must be true that $60x + 80y = 1,440$. The equations $x + y = 21$ and $60x + 80y = 1,440$ form a linear system of equations that can be solved to find the value of x , which represents the number of children in the group, using the elimination method. Multiplying both sides of the equation $x + y = 21$ by 80 yields $80x + 80y = 1,680$. Subtracting $60x + 80y = 1,440$ from $80x + 80y = 1,680$ yields $(80x + 80y) - (60x + 80y) = 1,680 - 1,440$, which is equivalent to $80x - 60x + 80y - 80y = 240$, or $20x = 240$. Dividing both sides of this equation by 20 yields $x = 12$. Therefore, 12 people in the group were children.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is the number of adults in the group, not the number of children in the group. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 19

Choice A is correct. The x -intercept of a graph in the xy -plane is the point on the graph where $y = 0$. It's given that function h is defined by $h(x) = 4x + 28$. Therefore, the equation representing the graph of $y = h(x)$ is $y = 4x + 28$. Substituting 0 for y in the equation $y = 4x + 28$ yields $0 = 4x + 28$. Subtracting 28 from both sides of this equation yields $-28 = 4x$. Dividing both sides of this equation by 4 yields $-7 = x$. Therefore, the x -intercept of the graph of $y = h(x)$ in the xy -plane is $(-7, 0)$. It's given that the x -intercept of the graph of $y = h(x)$ is $(a, 0)$. Therefore, $a = -7$. The y -intercept of a graph in the xy -plane is the point on the graph where $x = 0$. Substituting 0 for x in the equation $y = 4x + 28$ yields $y = 4(0) + 28$, or $y = 28$. Therefore, the y -intercept of the graph of $y = h(x)$ in the xy -plane is $(0, 28)$. It's given that the y -intercept of the graph of $y = h(x)$ is $(0, b)$. Therefore, $b = 28$. If $a = -7$ and $b = 28$, then the value of $a + b$ is $-7 + 28$, or 21.

Choice B is incorrect. This is the value of b , not $a + b$. *Choice C* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect. This is the value of $-a + b$, not $a + b$.

QUESTION 20

The correct answer is 8. Since each term of the given expression, $2x^3 + 42x^2 + 208x$, has a factor of $2x$, the expression can be rewritten as $2x(x^2) + 2x(21x) + 2x(104)$, or $2x(x^2 + 21x + 104)$. Since the values 8 and 13 have a sum of 21 and a product of 104, the expression $x^2 + 21x + 104$ can be factored as $(x + 8)(x + 13)$. Therefore, the given expression can be factored as $2x(x + 8)(x + 13)$. It follows that the factors of the given expression are 2, x , $x + 8$, and $x + 13$. Of these factors, only $x + 8$ and $x + 13$ are of the form $x + b$, where b is a positive constant. Therefore, the possible values of b are 8 and 13. Thus, the smallest possible value of b is 8.

QUESTION 21

The correct answer is $\frac{29}{2}$. According to the first equation in the given system, the value of y is -1.5 . Substituting -1.5 for y in the second equation in the given system yields $-1.5 = x^2 + 8x + a$. Adding 1.5 to both sides of this equation yields $0 = x^2 + 8x + a + 1.5$. If the given system has exactly one distinct real solution, it follows that $0 = x^2 + 8x + a + 1.5$ has exactly one distinct real solution. A quadratic equation in the form $0 = px^2 + qx + r$, where p , q , and r are constants, has exactly one distinct real solution if and only if the discriminant, $q^2 - 4pr$, is equal to 0. The equation $0 = x^2 + 8x + a + 1.5$ is in this form, where $p = 1$, $q = 8$, and $r = a + 1.5$. Therefore, the discriminant of the equation $0 = x^2 + 8x + a + 1.5$ is $(8)^2 - 4(1)(a + 1.5)$, or $64 - 4(a + 1.5)$, or $58 - 4a$. Setting the discriminant equal to 0 to solve for a yields $58 - 4a = 0$. Adding $4a$ to both sides of this equation yields $58 = 4a$. Dividing both sides of this equation by 4 yields $\frac{58}{4} = a$, or $\frac{29}{2} = a$. Therefore, if the given system of equations has exactly one distinct real solution, the value of a is $\frac{29}{2}$. Note that $29/2$ and 14.5 are examples of ways to enter a correct answer.

QUESTION 22

Choice B is correct. It's given that $f(x) = (x + 6)(x + 5)(x - 4)$ and $y = f(x) - 3$. Substituting $(x + 6)(x + 5)(x - 4)$ for $f(x)$ in the equation $y = f(x) - 3$ yields $y = (x + 6)(x + 5)(x - 4) - 3$. Substituting -6 for x in this equation yields $y = (-6 + 6)(-6 + 5)(-6 - 4) - 3$, or $y = -3$. Substituting -5 for x in the equation $y = (x + 6)(x + 5)(x - 4) - 3$ yields $y = (-5 + 6)(-5 + 5)(-5 - 4) - 3$, or $y = -3$. Substituting 4 for x in the equation $y = (x + 6)(x + 5)(x - 4) - 3$ yields $y = (4 + 6)(4 + 5)(4 - 4) - 3$, or $y = -3$. Therefore, when $x = -6$ then $y = -3$, when $x = -5$ then $y = -3$, and when $x = 4$ then $y = -3$. Thus, the table of values in choice B represents $y = f(x) - 3$.

Choice A is incorrect. This table represents $y = x - 3$ rather than $y = f(x) - 3$.

Choice C is incorrect. This table represents $y = x + 3$ rather than $y = f(x) - 3$.

Choice D is incorrect. This table represents $y = f(x) + 3$ rather than $y = f(x) - 3$.

QUESTION 23

Choice C is correct. Since the value of $q(x)$ decreases by a fixed percentage, 45%, for every increase in the value of x by 1, the function q is a decreasing exponential function. A decreasing exponential function can be written in the form

$q(x) = a\left(1 - \frac{p}{100}\right)^x$, where a is the value of $q(0)$ and the value of $q(x)$ decreases

by $p\%$ for every increase in the value of x by 1. If $q(0) = 14$, then $a = 14$. Since

the value of $q(x)$ decreases by 45% for every increase in the value of x by 1,

$p = 45$. Substituting 14 for a and 45 for p in the equation $q(x) = a\left(1 - \frac{p}{100}\right)^x$

yields $q(x) = 14\left(1 - \frac{45}{100}\right)^x$, which is equivalent to $q(x) = 14(1 - 0.45)^x$, or

$q(x) = 14(0.55)^x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. For this function, the value of $q(x)$ increases, rather than decreases, by 45% for every increase in the value of x by 1.

QUESTION 24

Choice A is correct. An equation for the graph shown can be written in slope-intercept form $y = mx + b$, where m is the slope of the graph and its y -intercept is $(0, b)$. Since the y -intercept of the graph shown is $(0, 2)$, the value of b is 2.

Since the graph also passes through the point $(4, 1)$, the slope can be calculated

as $\frac{1-2}{4-0}$, or $-\frac{1}{4}$. Therefore, the value of m is $-\frac{1}{4}$. Substituting $-\frac{1}{4}$ for m and 2

for b in the equation $y = mx + b$ yields $y = -\frac{1}{4}x + 2$. It's given that an equation

for the graph shown is $y = f(x) + 14$. Substituting $f(x) + 14$ for y in the equation

$y = -\frac{1}{4}x + 2$ yields $f(x) + 14 = -\frac{1}{4}x + 2$. Subtracting 14 from both sides of this

equation yields $f(x) = -\frac{1}{4}x - 12$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 25

Choice B is correct. It's given that right triangle RST is similar to triangle UVW , where S corresponds to V and T corresponds to W . It's given that the side lengths of the right triangle RST are $RS = 20$, $ST = 48$, and $TR = 52$.

Corresponding angles in similar triangles are equal. It follows that the measure of angle T is equal to the measure of angle W . The hypotenuse of a right triangle is

the longest side. It follows that the hypotenuse of triangle RST is side TR . The hypotenuse of a right triangle is the side opposite the right angle. Therefore, angle S is a right angle. The adjacent side of an acute angle in a right triangle is the side closest to the angle that is not the hypotenuse. It follows that the adjacent side of angle T is side ST . The opposite side of an acute angle in a right triangle is the side across from the acute angle. It follows that the opposite side of angle T is side RS . The tangent of an acute angle in a right triangle is the ratio of the length

of the opposite side to the length of the adjacent side. Therefore, $\tan T = \frac{RS}{ST}$.

Substituting 20 for RS and 48 for ST in this equation yields $\tan T = \frac{20}{48}$, or

$\tan T = \frac{5}{12}$. The tangents of two acute angles with equal measures are equal.

Since the measure of angle T is equal to the measure of angle W , it follows that

$\tan T = \tan W$. Substituting $\frac{5}{12}$ for $\tan T$ in this equation yields $\frac{5}{12} = \tan W$.

Therefore, the value of $\tan W$ is $\frac{5}{12}$.

Choice A is incorrect. This is the value of $\sin W$. *Choice C* is incorrect. This is the value of $\cos W$. *Choice D* is incorrect. This is the value of $\frac{1}{\tan W}$.

QUESTION 26

Choice A is correct. It's given that w represents the total wall area, in square feet.

Since the walls of the room will be painted twice, the amount of paint, in gallons, needs to cover $2w$ square feet. It's also given that one gallon of paint will cover 220 square feet. Dividing the total area, in square feet, of the surface to be painted by the number of square feet covered by one gallon of paint gives the number of gallons of paint that will be needed. Dividing $2w$ by 220 yields $\frac{2w}{220}$, or $\frac{w}{110}$. Therefore, the equation that represents the total amount of paint P , in gallons, needed to paint the walls of the room twice is $P = \frac{w}{110}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from finding the amount of paint needed to paint the walls once rather than twice. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 27

The correct answer is 9.87. It's given that the number a is 110% greater than the number b . It follows that $a = \left(1 + \frac{110}{100}\right)b$, or $a = 2.1b$. It's also given that the number b is 90% less than 47. It follows that $b = \left(1 - \frac{90}{100}\right)(47)$, or $b = 0.1(47)$, which yields $b = 4.7$. Substituting 4.7 for b in the equation $a = 2.1b$ yields $a = 2.1(4.7)$, which is equivalent to $a = 9.87$. Therefore, the value of a is 9.87.