

Math

Module 2

(27 questions)

QUESTION 1

Choice B is correct. The number of harvested potatoes Isabel saved to plant next year can be calculated by multiplying the total number of potatoes Isabel harvested, 760, by the proportion of potatoes she saved. Since she saved 10% of the potatoes she harvested, the proportion of potatoes she saved is $\frac{10}{100}$, or 0.1. Multiplying 760 by this proportion gives $760(0.1)$, or 76, potatoes that she saved to plant next year.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 2

Choice B is correct. The y -intercept of a graph in the xy -plane is the point at which the graph crosses the y -axis. The graph shown crosses the y -axis at the point $(0, 2)$. Therefore, the y -intercept of the graph shown is $(0, 2)$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 3

Choice C is correct. Since 1 meter is equal to 100 centimeters, 51 meters is equal to 51 meters $\left(\frac{100 \text{ centimeters}}{1 \text{ meter}}\right)$, or 5,100 centimeters.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from dividing, rather than multiplying, 51 by 100. *Choice D* is incorrect. This is the length, in millimeters rather than centimeters, that is equivalent to a length of 51 meters.

QUESTION 4

Choice C is correct. It's given that t represents the number of seconds after the bus passes the marker. Substituting 2 for t in the given equation $d = 30t$ yields $d = 30(2)$, or $d = 60$. Therefore, the bus will be 60 feet from the marker 2 seconds after passing it.

Choice A is incorrect. This is the distance, in feet, the bus will be from the marker 1 second, not 2 seconds, after passing it. **Choice B** is incorrect and may result from conceptual or calculation errors. **Choice D** is incorrect. This is the distance, in feet, the bus will be from the marker 3 seconds, not 2 seconds, after passing it.

QUESTION 5

Choice B is correct. Combining like terms inside the parentheses of the given expression, $20w - (4w + 3w)$, yields $20w - (7w)$. Combining like terms in this resulting expression yields $13w$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 6

The correct answer is 27. Multiplying both sides of the given equation by 3 yields $3(6 + x) = 3(9)$, or $18 + 3x = 27$. Therefore, the value of $18 + 3x$ is 27.

QUESTION 7

The correct answer is 7. When an equation is of the form $y = ax^2 + bx + c$, where a , b , and c are constants, the value of y reaches its minimum when $x = -\frac{b}{2a}$.

Since the given equation is of the form $y = ax^2 + bx + c$, it follows that $a = 1$,

$b = -14$, and $c = 22$. Therefore, the value of y reaches its minimum when

$x = -\frac{(-14)}{2(1)}$, or $x = 7$.

QUESTION 8

Choice A is correct. Since x is a factor of each term in the given expression, the expression is equivalent to $x(9x) + x(5)$, or $x(9x + 5)$.

Choice B is incorrect. This expression is equivalent to $45x^2 + 5x$, not $9x^2 + 5x$.

Choice C is incorrect. This expression is equivalent to $9x^2 + 45x$, not $9x^2 + 5x$.

Choice D is incorrect. This expression is equivalent to $9x^3 + 5x^2$, not $9x^2 + 5x$.

QUESTION 9

Choice D is correct. The sum of the angle measures of a triangle is 180° . Adding the measures of angles B and C gives $52 + 17 = 69^\circ$. Therefore, the measure of angle A is $180 - 69 = 111^\circ$.

Choice A is incorrect and may result from subtracting the sum of the measures of angles B and C from 90° , instead of from 180° . *Choice B* is incorrect and may result from subtracting the measure of angle C from the measure of angle B . *Choice C* is incorrect and may result from adding the measures of angles B and C but not subtracting the result from 180° .

QUESTION 10

Choice D is correct. Since the graphs of the equations in the given system intersect at the point (x, y) , the point (x, y) represents a solution to the given system of equations. The first equation of the given system of equations states that $x = 8$. Substituting 8 for x in the second equation of the given system of equations yields $y = 8^2 + 8$, or $y = 72$. Therefore, the value of y is 72.

Choice A is incorrect. This is the value of x , not y . *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice C* is incorrect and may result from conceptual or calculation errors.

QUESTION 11

Choice B is correct. The line of best fit shown intersects the y -axis at a positive y -value and has a negative slope. The graph of an equation of the form $y = a + bx$, where a and b are constants, intersects the y -axis at a y -value of a and has a slope of b . Of the given choices, only choice B represents a line that intersects the y -axis at a positive y -value, 13.5, and has a negative slope, -0.8 .

Choice A is incorrect. This equation represents a line that has a positive slope, not a negative slope. *Choice C* is incorrect. This equation represents a line that intersects the y -axis at a negative y -value, not a positive y -value, and has a positive slope, not a negative slope. *Choice D* is incorrect. This equation represents a line that intersects the y -axis at a negative y -value, not a positive y -value.

QUESTION 12

Choice C is correct. It's given that $f(x) = 8\sqrt{x}$. Substituting 48 for $f(x)$ in this equation yields $48 = 8\sqrt{x}$. Dividing both sides of this equation by 8 yields $6 = \sqrt{x}$. This can be rewritten as $\sqrt{x} = 6$. Squaring both sides of this equation yields $x = 36$. Therefore, the value of x for which $f(x) = 48$ is 36.

Choice A is incorrect. If $x = 6$, $f(x) = 8\sqrt{6}$, not 48. *Choice B* is incorrect. If $x = 8$, $f(x) = 8\sqrt{8}$, not 48. *Choice D* is incorrect. If $x = 64$, $f(x) = 8\sqrt{64}$, which is equivalent to 64, not 48.

QUESTION 13

The correct answer is 46. It's given that O is the center of a circle and that points R and S lie on the circle. Therefore, $\overline{OR}$ and $\overline{OS}$ are radii of the circle. It follows that $OR = OS$. If two sides of a triangle are congruent, then the angles opposite them are congruent. It follows that the angles $\angle RSO$ and $\angle ORS$, which are across from the sides of equal length, are congruent. Let x° represent the

measure of $\angle RSO$. It follows that the measure of $\angle ORS$ is also x° . It's given that the measure of $\angle ROS$ is 88° . Because the sum of the measures of the interior angles of a triangle is 180° , the equation $x^\circ + x^\circ + 88^\circ = 180^\circ$, or $2x + 88 = 180$, can be used to find the measure of $\angle RSO$. Subtracting 88 from both sides of this equation yields $2x = 92$. Dividing both sides of this equation by 2 yields $x = 46$. Therefore, the measure of $\angle RSO$, in degrees, is 46.

QUESTION 14

The correct answer is $\frac{29}{3}$. Applying the distributive property to the left-hand side of the given equation, $x(x+1) - 56$, yields $x^2 + x - 56$. Applying the distributive property to the right-hand side of the given equation, $4x(x-7)$, yields $4x^2 - 28x$. Thus, the equation becomes $x^2 + x - 56 = 4x^2 - 28x$. Combining like terms on the left- and right-hand sides of this equation yields $0 = (4x^2 - x^2) + (-28x - x) + 56$, or $3x^2 - 29x + 56 = 0$. For a quadratic equation in the form $ax^2 + bx + c = 0$, where a , b , and c are constants, the quadratic formula gives the solutions to

the equation in the form $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Substituting 3 for a , -29 for b , and

56 for c from the equation $3x^2 - 29x + 56 = 0$ into the quadratic formula yields

$$x = \frac{29 \pm \sqrt{(-29)^2 - 4(3)(56)}}{2(3)}, \text{ or } x = \frac{29}{6} \pm \frac{13}{6}.$$

It follows that the solutions to the given

equation are $\frac{29}{6} + \frac{13}{6}$ and $\frac{29}{6} - \frac{13}{6}$. Adding these two solutions gives the sum of the

solutions: $\frac{29}{6} + \frac{13}{6} + \frac{29}{6} - \frac{13}{6}$, which is equivalent to $\frac{29}{6} + \frac{29}{6}$, or $\frac{29}{3}$. Note that 29/3,

9.666, and 9.667 are examples of ways to enter a correct answer.

QUESTION 15

Choice C is correct. It's given by the first equation in the system that $y = 3x$.

Substituting $3x$ for y in the equation $2x + y = 12$ yields $2x + 3x = 12$, or $5x = 12$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 16

Choice D is correct. The volume, V , of a cube can be found using the formula $V = s^3$, where s is the edge length of the cube. It's given that a cube has an edge length of 41 inches. Substituting 41 inches for s in this equation yields $V = 41^3$ cubic inches, or $V = 68,921$ cubic inches. Therefore, the volume of the cube is 68,921 cubic inches.

Choice A is incorrect. This is the perimeter, in inches, of the cube. *Choice B* is incorrect. This is the area, in square inches, of a face of the cube. *Choice C* is incorrect. This is the surface area, in square inches, of the cube.

QUESTION 17

Choice D is correct. It's given that the function p models the population of Lowell t years after a census. Since there are 12 months in a year, m months after the census is equivalent to $\frac{m}{12}$ years after the census. Substituting $\frac{m}{12}$ for t in the equation $p(t) = 90,000(1.06)^t$ yields $p\left(\frac{m}{12}\right) = 90,000(1.06)^{\frac{m}{12}}$. Therefore, the function r that best models the population of Lowell m months after the census is $r(m) = 90,000(1.06)^{\frac{m}{12}}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 18

Choice A is correct. The given system of linear equations can be solved by the elimination method. Multiplying each side of the second equation in the given system by 3 yields $(2x + 2y)(3) = (10)(3)$, or $6x + 6y = 30$. Subtracting this equation from the first equation in the given system yields $(6x + 7y) - (6x + 6y) = (28) - (30)$, which is equivalent to $(6x - 6x) + (7y - 6y) = 28 - 30$, or $y = -2$.

Choice B is incorrect. This is the value of x , not the value of y . **Choice C** is incorrect and may result from conceptual or calculation errors. **Choice D** is incorrect and may result from conceptual or calculation errors.

QUESTION 19

Choice B is correct. It's given that the minimum value of x is 12 less than 6 times another number n . Therefore, the possible values of x are all greater than or equal to the value of 12 less than 6 times n . The value of 6 times n is given by the expression $6n$. The value of 12 less than $6n$ is given by the expression $6n - 12$. Therefore, the possible values of x are all greater than or equal to $6n - 12$. This can be shown by the inequality $x \geq 6n - 12$.

Choice A is incorrect. This inequality shows the possible values of x if the maximum, not the minimum, value of x is 12 less than 6 times n . **Choice C** is incorrect. This inequality shows the possible values of x if the maximum, not the minimum, value of x is 6 times n less than 12, not 12 less than 6 times n .

Choice D is incorrect. This inequality shows the possible values of x if the minimum value of x is 6 times n less than 12, not 12 less than 6 times n .

QUESTION 20

The correct answer is 44. The mean of a data set is computed by dividing the sum of the values in the data set by the number of values in the data set. It's given that data set A consists of the heights of 75 buildings and has a mean of 32 meters.

This can be represented by the equation $\frac{x}{75} = 32$, where x represents the sum of the heights of the buildings, in meters, in data set A. Multiplying both sides of this equation by 75 yields $x = 75(32)$, or $x = 2,400$ meters. Therefore, the sum

of the heights of the buildings in data set A is 2,400 meters. It's also given that data set B consists of the heights of 50 buildings and has a mean of 62 meters. This can be represented by the equation $\frac{y}{50} = 62$, where y represents the sum of the heights of the buildings, in meters, in data set B. Multiplying both sides of this equation by 50 yields $y = 50(62)$, or $y = 3,100$ meters. Therefore, the sum of the heights of the buildings in data set B is 3,100 meters. Since it's given that data set C consists of the heights of the 125 buildings from data sets A and B, it follows that the mean of data set C is the sum of the heights of the buildings, in meters, in data sets A and B divided by the number of buildings represented in data sets A and B, or $\frac{2,400+3,100}{125}$, which is equivalent to 44 meters. Therefore, the mean, in meters, of data set C is 44.

QUESTION 21

The correct answer is $\frac{59}{9}$. When the graph of an equation in the form $Ax + By = C$, where A , B , and C are constants, is translated down k units in the xy -plane, the resulting graph can be represented by the equation $Ax + B(y + k) = C$. It's given that the graph of $9x - 10y = 19$ is translated down 4 units in the xy -plane. Therefore, the resulting graph can be represented by the equation $9x - 10(y + 4) = 19$, or $9x - 10y - 40 = 19$. Adding 40 to both sides of this equation yields $9x - 10y = 59$. The x -coordinate of the x -intercept of the graph of an equation in the xy -plane is the value of x in the equation when $y = 0$. Substituting 0 for y in the equation $9x - 10y = 59$ yields $9x - 10(0) = 59$, or $9x = 59$. Dividing both sides of this equation by 9 yields $x = \frac{59}{9}$. Therefore, the x -coordinate of the x -intercept of the resulting graph is $\frac{59}{9}$. Note that 59/9, 6.555, and 6.556 are examples of ways to enter a correct answer.

QUESTION 22

Choice D is correct. Since the value of y increases by a constant factor, 4, for each increase of 1 in the value of x , the relationship between x and y is exponential. An exponential relationship between x and y can be represented by an equation of the form $y = a(b)^x$, where a is the value of x when $y = 0$ and y increases by a factor of b for each increase of 1 in the value of x . Since $y = 200$ when $x = 0$, $a = 200$. Since y increases by a factor of 4 for each increase of 1 in the value of x , $b = 4$. Substituting 200 for a and 4 for b in the equation $y = a(b)^x$ yields $y = 200(4)^x$. Thus, the equation $y = 200(4)^x$ represents the relationship between x and y .

Choice A is incorrect and may result from conceptual errors. **Choice B** is incorrect. This equation represents a relationship where for each increase of 1 in the value of x , the value of y increases by a factor of 200, not 4, and when $x = 0$, y is equal to 4, not 200. **Choice C** is incorrect and may result from conceptual errors.

QUESTION 23

Choice B is correct. Adding 9 to each side of the given equation yields $x^2 - 2x = 9$. To complete the square, adding 1 to each side of this equation

yields $x^2 - 2x + 1 = 9 + 1$, or $(x - 1)^2 = 10$. Taking the square root of each side of this equation yields $x - 1 = \pm\sqrt{10}$. Adding 1 to each side of this equation yields $x = 1 \pm \sqrt{10}$. Since it's given that one of the solutions to the equation can be written as $1 + \sqrt{k}$, the value of k must be 10.

Alternate approach: It's given that $1 + \sqrt{k}$ is a solution to the given equation. It follows that $x = 1 + \sqrt{k}$. Substituting $1 + \sqrt{k}$ for x in the given equation yields $(1 + \sqrt{k})^2 - 2(1 + \sqrt{k}) - 9 = 0$, or $(1 + \sqrt{k})(1 + \sqrt{k}) - 2(1 + \sqrt{k}) - 9 = 0$. Expanding the products on the left-hand side of this equation yields $1 + 2\sqrt{k} + k - 2 - 2\sqrt{k} - 9 = 0$, or $k - 10 = 0$. Adding 10 to each side of this equation yields $k = 10$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 24

Choice A is correct. The median of a data set with an odd number of values that are in ascending or descending order is the middle value of the data set. Since the distribution of the values of both data set A and data set B form symmetric dot plots, and each data set has an odd number of values, it follows that the median is given by the middle value in each of the dot plots. Thus, the median of data set A is 13, and the median of data set B is 13. Therefore, statement I is true. Data set A and data set B have the same frequency for each of the values 11, 12, 14, and 15. Data set A has a frequency of 1 for values 10 and 16, whereas data set B has a frequency of 2 for values 10 and 16. Standard deviation is a measure of the spread of a data set; it is larger when there are more values further from the mean, and smaller when there are more values closer to the mean. Since both distributions are symmetric with an odd number of values, the mean of each data set is equal to its median. Thus, each data set has a mean of 13. Since more of the values in data set A are closer to 13 than data set B, it follows that data set A has a smaller standard deviation than data set B. Thus, statement II is false. Therefore, only statement I must be true.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 25

Choice B is correct. It's given that the right triangle is isosceles. In an isosceles right triangle, the two legs have equal lengths, and the length of the hypotenuse is $\sqrt{2}$ times the length of one of the legs. Let ℓ represent the length, in inches, of each leg of the isosceles right triangle. It follows that the length of the hypotenuse is $\ell\sqrt{2}$ inches. The perimeter of a figure is the sum of the lengths of the sides of the figure. Therefore, the perimeter of the isosceles right triangle is $\ell + \ell + \ell\sqrt{2}$ inches. It's given that the perimeter of the triangle is $94 + 94\sqrt{2}$ inches. It follows that $\ell + \ell + \ell\sqrt{2} = 94 + 94\sqrt{2}$. Factoring the left-hand side of this equation yields $(1 + 1 + \sqrt{2})\ell = 94 + 94\sqrt{2}$, or $(2 + \sqrt{2})\ell = 94 + 94\sqrt{2}$. Dividing both sides of this

equation by $2 + \sqrt{2}$ yields $\ell = \frac{94 + 94\sqrt{2}}{2 + \sqrt{2}}$. Rationalizing the denominator of the right-hand side of this equation by multiplying the right-hand side of the equation by $\frac{2 - \sqrt{2}}{2 - \sqrt{2}}$ yields $\ell = \frac{(94 + 94\sqrt{2})(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}$. Applying the distributive property to the numerator and to the denominator of the right-hand side of this equation yields $\ell = \frac{188 - 94\sqrt{2} + 188\sqrt{2} - 94\sqrt{4}}{4 - 2\sqrt{2} + 2\sqrt{2} - \sqrt{4}}$. This is equivalent to $\ell = \frac{94\sqrt{2}}{2}$, or $\ell = 47\sqrt{2}$. Therefore, the length, in inches, of one leg of the isosceles right triangle is $47\sqrt{2}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the length, in inches, of the hypotenuse. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 26

Choice C is correct. It's given that the equation $-9x^2 + 30x + c = 0$ has exactly one solution. A quadratic equation of the form $ax^2 + bx + c = 0$ has exactly one solution if and only if its discriminant, $-4ac + b^2$, is equal to zero. It follows that for the given equation, $a = -9$ and $b = 30$. Substituting -9 for a and 30 for b into $b^2 - 4ac$ yields $30^2 - 4(-9)(c)$, or $900 + 36c$. Since the discriminant must equal zero, $900 + 36c = 0$. Subtracting $36c$ from both sides of this equation yields $900 = -36c$. Dividing each side of this equation by -36 yields $-25 = c$. Therefore, the value of c is -25 .

Choice A is incorrect. If the value of c is 3 , this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution. *Choice B* is incorrect. If the value of c is 0 , this would yield a discriminant that is greater than zero. Therefore, the given equation would have two solutions, rather than exactly one solution. *Choice D* is incorrect. If the value of c is -53 , this would yield a discriminant that is less than zero. Therefore, the given equation would have no real solutions, rather than exactly one solution.

QUESTION 27

The correct answer is **6**. A system of two linear equations in two variables, x and y , has no solution if the lines represented by the equations in the xy -plane are parallel and distinct. Lines represented by equations in standard form, $Ax + By = C$ and $Dx + Ey = F$, are parallel if the coefficients for x and y in one equation are proportional to the corresponding coefficients in the other

equation, meaning $\frac{D}{A} = \frac{E}{B}$; and the lines are distinct if the constants are not proportional, meaning $\frac{F}{C}$ is not equal to $\frac{D}{A}$ or $\frac{E}{B}$. The first equation in the given system is $\frac{3}{2}y - \frac{1}{4}x = \frac{2}{3} - \frac{3}{2}y$. Multiplying each side of this equation by 12 yields $18y - 3x = 8 - 18y$. Adding $18y$ to each side of this equation yields $36y - 3x = 8$, or $-3x + 36y = 8$. The second equation in the given system is $\frac{1}{2}x + \frac{3}{2} = py + \frac{9}{2}$.

Multiplying each side of this equation by 2 yields $x + 3 = 2py + 9$. Subtracting $2py$ from each side of this equation yields $x + 3 - 2py = 9$. Subtracting 3 from each side of this equation yields $x - 2py = 6$. Therefore, the two equations in the given system, written in standard form, are $-3x + 36y = 8$ and $x - 2py = 6$. As previously stated, if this system has no solution, the lines represented by the equations in the xy -plane are parallel and distinct, meaning the proportion

$\frac{1}{-3} = \frac{-2p}{36}$, or $-\frac{1}{3} = -\frac{p}{18}$, is true and the proportion $\frac{6}{8} = \frac{1}{-3}$ is not true. The proportion $\frac{6}{8} = \frac{1}{-3}$ is not true. Multiplying each side of the true proportion, $-\frac{1}{3} = -\frac{p}{18}$, by -18 yields $6 = p$. Therefore, if the system has no solution, then the value of p is 6.