Math

Module 1
(27 questions)

QUESTION 1

Choice C is correct. The solution to the system of two equations corresponds to the point where the graphs of the equations intersect. The graphs of the linear equation and the nonlinear equation shown intersect at the point $(4, 5)$. Thus, the solution to the system is $(4, 5)$.

Choice A is incorrect and may result from conceptual or calculation errors. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 2

Choice B is correct. It's given that the film club has 90 members on the first day of a semester, and 10 new members join the film club each day after the first day of the semester. This means that after 4 days, 4×10, or 40, new members will have joined the club. Adding 40 members to the original 90 club members yields 130 members. Thus, the film club will have 130 total members 4 days after the first day of the semester.

Choice A is incorrect. This is the number of members that will have joined the film club 4 days after the first day of the semester if 100 new members, not 10, join the film club each day. *Choice C* is incorrect. This is the number of members the film club will have 4 days after the first day of the semester if 1 new member, not 10, joins the film club each day. *Choice D* is incorrect. This is the number of members the film club has on the first day of the semester.

QUESTION 3

Choice D is correct. The y-intercept of a graph is the point where the graph intersects the y-axis. The graph of function f shown intersects the y-axis at the point $(0, 8)$. Therefore, the y-intercept of the graph of f is $(0, 8)$.

Choice A is incorrect. This is the point where the x-axis, not the graph of f, intersects the y-axis. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice C* is incorrect and may result from conceptual or calculation errors.

QUESTION 4

Choice B is correct. The second equation in the given system is $r = 3$. Substituting 3 for r in the first equation in the given system yields $s + 7(3) = 27$, or $s + 21 = 27$. Subtracting 21 from both sides of this equation yields $s = 6$. Therefore, the solution (r, s) to the given system of equations is $(3, 6)$.

Choice A is incorrect. This is the solution (s, r), not (r, s), to the given system of equations. *Choice C* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 5

Choice B is correct. The given values show that as x increases, $f(x)$ also increases, which means that f is an increasing function. Furthermore, $f(x)$ increases at a constant rate of 1 for each increase of x by 1. A function with a constant rate of change is linear. Thus, the function f can be described as an increasing linear function.

Choice A is incorrect. For a decreasing linear function, as x increases, $f(x)$ decreases rather than increases. *Choice C* is incorrect. For a decreasing exponential function, for each increase of x by 1, $f(x)$ decreases by a fixed percentage rather than increases at a constant rate. *Choice D* is incorrect. For an increasing exponential function, for each increase of x by 1, $f(x)$ increases by a fixed percentage rather than at a constant rate.

QUESTION 6

The correct answer is 4. A solution to a system of equations must satisfy each equation in the system. It follows that if (x, y) is a solution to the system, the point (x, y) lies on the graph in the xy-plane of each equation in the system. According to the graph, the point (x, y) that lies on the graph of each equation in the system is $(4, 1)$. Therefore, the solution to the system is $(4, 1)$. It follows that the value of x is 4.

QUESTION 7

The correct answer is 29. The range of a data set is the difference between its maximum value and its minimum value. For the data set shown, the maximum score is 52 and the minimum score is 23. The difference between those scores is $52 - 23$, or 29. Therefore, the range of the 7 scores shown is 29.

QUESTION 8

Choice C is correct. Vertical angles, or angles that are opposite each other when two lines intersect, are congruent. It's given that line k intersects line n. Based on the figure, the angle with measure $x°$ and the angle with measure 145° are vertical angles. Therefore, the value of x is equal to 145.

Choice A is incorrect and may result from conceptual or calculation errors. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 9

Choice B is correct. It's given that the equation $x+y=1{,}440$ represents the number of minutes of daylight, x, and the number of minutes of non-daylight, y, on a particular day in Oak Park, Illinois. It's also given that this day has 670 minutes of daylight. Substituting 670 for x in the equation $x+y=1{,}440$ yields $670+y=1{,}440$. Subtracting 670 from both sides of this equation yields $y=770$. Therefore, this day has 770 minutes of non-daylight.

Choice A is incorrect. This is the number of minutes of daylight, not non-daylight, on this day. *Choice C* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect. This is the total number of minutes of daylight and non-daylight.

QUESTION 10

Choice B is correct. It's given that from the sample of 20 employees at the company, 16 of the employees are enrolled in exactly three professional development courses this year. Since $\left(\frac{16}{20}\right)$ is equal to 0.80, or $\frac{80}{100}$, it follows that 80% of the employees in the sample are enrolled in exactly three professional development courses this year. Therefore, the best estimate for the percentage of employees at the company who are enrolled in exactly three professional development courses this year is 80%. It's given that there are a total of 400 employees at the company. Therefore, the best estimate of the number of employees at the company who are enrolled in exactly three professional development courses this year is $\left(\frac{80}{100}\right)\left(400\right)$, or 320.

Choice A is incorrect. This is the number of employees from the sample who aren't enrolled in exactly three professional development courses this year. *Choice C* is incorrect. This is the number of employees who weren't selected for the sample. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 11

Choice C is correct. Dividing all terms in the given equation by 4 yields $\frac{4x}{4}-\frac{28}{4}=-\frac{24}{4}$, or $x-7=-6$. Therefore, the value of $x-7$ is -6.

Choice A is incorrect. This is the value of $4x-28$, not $x-7$. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from conceptual or calculation errors.

QUESTION 12

Choice D is correct. It's given that for a snowstorm in a certain town, the minimum rate of snowfall recorded was 0.6 inches per hour, the maximum rate of snowfall recorded was 1.8 inches per hour, and s represents a rate of snowfall, in inches per hour, recorded for this snowstorm. It follows that the inequality $0.6 \leq s \leq 1.8$ is true for all values of s.

Choice A is incorrect and may result from conceptual or calculation errors. *Choice B* is incorrect and may result from conceptual or calculation errors. *Choice C* is incorrect and may result from conceptual or calculation errors.

QUESTION 13

The correct answer is 6. It's given that $y = 4x$ and $y = x^2 - 12$. Since $y = 4x$, substituting $4x$ for y in the second equation of the given system yields $4x = x^2 - 12$. Subtracting $4x$ from both sides of this equation yields $0 = x^2 - 4x - 12$. This equation can be rewritten as $0 = (x - 6)(x + 2)$. By the zero product property, $x - 6 = 0$ or $x + 2 = 0$. Adding 6 to both sides of the equation $x - 6 = 0$ yields $x = 6$. Subtracting 2 from both sides of the equation $x + 2 = 0$ yields $x = -2$. Therefore, solutions to the given system of equations occur when $x = 6$ and when $x = -2$. It's given that a solution to the given system of equations is (x, y), where $x > 0$. Since 6 is greater than 0, it follows that the value of x is 6.

QUESTION 14

The correct answer is 4.51. It's given that the equation $4.51x + 6.07y = 896.86$ represents this situation, where x is the number of smaller containers sold, y is the number of larger containers sold, and 896.86 is the store's total sales, in dollars, of blueberries last month. Therefore, $4.51x$ represents the store's sales, in dollars, of smaller containers, and $6.07y$ represents the store's sales, in dollars, of larger containers. Since x is the number of smaller containers sold, the price, in dollars, of each smaller container is 4.51.

QUESTION 15

Choice C is correct. The volume, V, of a right circular cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. It's given that a right circular cylinder has a height of 6 centimeters. Therefore, $h = 6$. It's also given that the cylinder has a base diameter of 22 centimeters. The radius of a circle is half the diameter of the circle. Since the base of a right circular cylinder is a circle, it follows that the radius of the base of the right circular cylinder is $\frac{22}{2}$, or 11, centimeters. Therefore, $r = 11$. Substituting 11 for r and 6 for h in the formula $V = \pi r^2 h$ yields $V = \pi(11)^2(6)$, which is equivalent to $V = \pi(121)(6)$, or $V = 726\pi$. Therefore, the volume, in cubic centimeters, of the cylinder is 726π.

Choice A is incorrect. This is the volume of a right circular cylinder that has a base diameter of $2\sqrt{22}$, not 22, centimeters and a height of 6 centimeters. *Choice B* is incorrect. This is the volume of a right circular cylinder that has a base diameter of $4\sqrt{11}$, not 22, centimeters and a height of 6 centimeters. *Choice D* is incorrect. This is the volume of a right circular cylinder that has a base diameter of 44, not 22, centimeters and a height of 6 centimeters.

QUESTION 16

Choice D is correct. It's given that the graph of the rational function f is shown, where $y = f(x)$ and $x \geq 0$. The graph shown passes through the point $(3, 3)$. It follows that when the value of x is 3, the value of $f(x)$ is 3. When the value of $f(x)$ is 3, the value of $f(x) + 5$ is $3 + 5$, or 8. Therefore, the graph of $y = f(x) + 5$ passes through the point $(3, 8)$. Of the given choices, choice D is the only graph that passes through the point $(3, 8)$ and is therefore the graph of $y = f(x) + 5$.

Choice A is incorrect. This is the graph of $y = f(x) - 5$, rather than $y = f(x) + 5$.
Choice B is incorrect. This is the graph of $y = \frac{f(x)}{5}$, rather than $y = f(x) + 5$.
Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 17

Choice B is correct. It's given that at a particular track meet, the ratio of coaches to athletes is 1 to 26. If one number in a ratio is multiplied by a value, the other number must be multiplied by the same value in order to maintain the same ratio. If there are x coaches at the track meet, multiplying both numbers in the ratio by x yields $1(x)$ to $26(x)$, or x to $26x$. Therefore, the expression $26x$ represents the number of athletes at the track meet.

Choice A is incorrect and may result from conceptual or calculation errors.
Choice C is incorrect and may result from conceptual or calculation errors.
Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 18

Choice D is correct. It's given that the equation $y - 5x = 6$ represents the relationship between the number of suits that Kaylani made, x, and the total length of fabric she purchased, y, in yards. Adding $5x$ to both sides of the given equation yields $y = 5x + 6$. Since Kaylani made x suits and used 5 yards of fabric to make each suit, the expression $5x$ represents the total amount of fabric she used to make the suits. Since y represents the total length of fabric Kaylani purchased, in yards, it follows from the equation $y = 5x + 6$ that Kaylani purchased $5x$ yards of fabric to make the suits, plus an additional 6 yards of fabric. Therefore, the best interpretation of 6 in this context is that Kaylani purchased 6 yards more fabric than she used to make the suits.

Choice A is incorrect. Kaylani made a total of x suits, not 6 suits. *Choice B* is incorrect. Kaylani purchased a total of y yards of fabric, not a total of 6 yards of fabric. *Choice C* is incorrect. Kaylani used a total of $5x$ yards of fabric to make the suits, not a total of 6 yards of fabric.

QUESTION 19

Choice A is correct. A trigonometric ratio can be found using the unit circle, that is, a circle with radius 1 unit. If a central angle of a unit circle in the xy-plane centered at the origin has its starting side on the positive x-axis and its terminal side intersects the circle at a point (x, y), then the value of the tangent of the

central angle is equal to the *y*-coordinate divided by the *x*-coordinate. There are 2π radians in a circle. Dividing $\frac{92\pi}{3}$ by 2π yields $\frac{92}{6}$, which is equivalent to $15+\frac{2}{3}$. It follows that on the unit circle centered at the origin in the *xy*-plane, the angle $\frac{92\pi}{3}$ is the result of 15 revolutions from its starting side on the positive *x*-axis followed by a rotation through $\frac{2\pi}{3}$ radians. Therefore, the angles $\frac{92\pi}{3}$ and $\frac{2\pi}{3}$ are coterminal angles and $\tan\left(\frac{92\pi}{3}\right)$ is equal to $\tan\left(\frac{2\pi}{3}\right)$. Since $\frac{2\pi}{3}$ is greater than $\frac{\pi}{2}$ and less than π, it follows that the terminal side of the angle is in quadrant II and forms an angle of $\frac{\pi}{3}$, or 60°, with the negative *x*-axis. Therefore, the terminal side of the angle intersects the unit circle at the point $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$. It follows that the value of $\tan\left(\frac{2\pi}{3}\right)$ is $\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$, which is equivalent to $-\sqrt{3}$. Therefore, the value of $\tan\left(\frac{92\pi}{3}\right)$ is $-\sqrt{3}$.

Choice B is incorrect. This is the value of $\frac{1}{\tan\left(\frac{92\pi}{3}\right)}$, not $\tan\left(\frac{92\pi}{3}\right)$. *Choice C* is incorrect. This is the value of $\frac{1}{\tan\left(\frac{\pi}{3}\right)}$, not $\tan\left(\frac{92\pi}{3}\right)$. *Choice D* is incorrect. This is the value of $\tan\left(\frac{\pi}{3}\right)$, not $\tan\left(\frac{92\pi}{3}\right)$.

QUESTION 20

The correct answer is $\frac{11}{28}$. The cosine of an acute angle in a right triangle is defined as the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse. In the triangle shown, the length of the leg adjacent to the angle with measure $x°$ is 11 units and the length of the hypotenuse is 28 units. Therefore, the value of $\cos x°$ is $\frac{11}{28}$. Note that 11/28, .3928, .3929, 0.392, and 0.393 are examples of ways to enter a correct answer.

QUESTION 21

The correct answer is 336. By the zero product property, if $(x+14)(t-x)=0$, then $x+14=0$, which gives $x=-14$, or $(t-x)=0$, which gives $x=t$. Therefore, $g(x)=0$ when $x=-14$ and when $x=t$. Since the graph of $y=g(x)$ passes through the point $(24,0)$, it follows that $g(24)=0$, so $t=24$. Substituting 24 for t in the equation $g(x)=(x+14)(t-x)$ yields $g(x)=(x+14)(24-x)$. The value of $g(0)$ can be calculated by substituting 0 for x in this equation, which yields $g(0)=(0+14)(24-0)$, or $g(0)=336$.

QUESTION 22

Choice B is correct. An equation of the form $(x-h)^2+(y-k)^2=r^2$, where h, k, and r are constants, represents a circle in the *xy*-plane with center (h, k) and radius r. Therefore, the circle represented by the given equation has center $(-4, 19)$ and radius 11. Since the center of the circle has an *x*-coordinate of -4 and the radius of the circle is 11, the least possible *x*-coordinate for any point on the circle is $-4-11$, or -15. Similarly, the greatest possible *x*-coordinate for any point on the circle is $-4+11$, or 7. Therefore, if the point (a, b) lies on the circle, it must be true that $-15\le a\le 7$. Of the given choices, only -14 satisfies this inequality.

Choice A is incorrect and may result from conceptual or calculation errors.
Choice C is incorrect and may result from conceptual or calculation errors.
Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 23

Choice D is correct. The volume of a right rectangular prism can be represented by a function V that gives the volume of the prism, in cubic inches, in terms of the length of the prism's base. The volume of a right rectangular prism is equal to the area of its base times its height. It's given that the length of the prism's base is x inches, which is 7 inches more than the width of the prism's base. This means that the width of the prism's base is $x-7$ inches. It follows that the area of the prism's base, in square inches, is $x(x-7)$ and the volume, in cubic inches, of the prism is $x(x-7)(9)$. Thus, the function V that gives the volume of this right rectangular prism, in cubic inches, in terms of the length of the prism's base, x, is $V(x)=9x(x-7)$.

Choice A is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base and the width of the base were 7 inches more than its length. *Choice B* is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base. *Choice C* is incorrect. This function would give the volume of the prism if the width of the base were 7 inches more than its length, rather than the length of the base being 7 inches more than its width.

QUESTION 24

Choice D is correct. A function of the form $f(x)=a(b)^x+c$, where $a<0$ and $b>1$, is a decreasing function. Both of the given functions are of this form; therefore, both are decreasing functions. If a function f is decreasing, as the value of x increases, the corresponding value of $f(x)$ decreases; therefore, the function doesn't have a minimum value. Thus, neither of the given functions has a minimum value.

Choice A is incorrect and may result from conceptual or calculation errors.
Choice B is incorrect and may result from conceptual or calculation errors.
Choice C is incorrect and may result from conceptual or calculation errors.

QUESTION 25

Choice A is correct. It's given that the result of increasing the quantity x by 400% is 60. This can be written as $x+\left(\frac{400}{100}\right)x=60$, which is equivalent to $x+4x=60$, or $5x=60$. Dividing each side of this equation by 5 yields $x=12$. Therefore, the value of x is 12.

Choice B is incorrect. The result of increasing the quantity 15 by 400% is 75, not 60. *Choice C* is incorrect. The result of increasing the quantity 240 by 400% is 1,200, not 60. *Choice D* is incorrect. The result of increasing the quantity 340 by 400% is 1,700, not 60.

QUESTION 26

Choice A is correct. It's given that the graph of $y=f(x)$ in the xy-plane passes through the points $(7, 0)$ and $(-3, 0)$. It follows that when the value of x is either 7 or -3, the value of $f(x)$ is 0. It's also given that the function f is defined by $f(x)=ax^2+bx+c$, where a, b, and c are constants. It follows that the function f is a quadratic function and, therefore, may be written in factored form as $f(x)=a(x-u)(x-v)$, where the value of $f(x)$ is 0 when x is either u or v. Since the value of $f(x)$ is 0 when the value of x is either 7 or -3, and the value of $f(x)$ is 0 when the value of x is either u or v, it follows that u and v are equal to 7 and -3. Substituting 7 for u and -3 for v in the equation $f(x)=a(x-u)(x-v)$ yields $f(x)=a(x-7)(x-(-3))$, or $f(x)=a(x-7)(x+3)$. Distributing the right-hand side of this equation yields $f(x)=a(x^2-7x+3x-21)$, or $f(x)=ax^2-4ax-21a$. Since it's given that $f(x)=ax^2+bx+c$, it follows that $b=-4a$. Adding a to each side of this equation yields $a+b=-3a$. Since $a+b=-3a$, if a is an integer, the value of $a+b$ must be a multiple of 3. If a is an integer greater than 1, it follows that $a\geq 2$. Therefore, $-3a\leq -3(2)$. It follows that the value of $a+b$ is less than or equal to $-3(2)$, or -6. Of the given choices, only -6 is a multiple of 3 that's less than or equal to -6.

Choice B is incorrect. This is the value of $a + b$ if a is equal to, not greater than, 1.
Choice C is incorrect and may result from conceptual or calculation errors.
Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 27

The correct answer is 25. The value of $g(7-w)$ is the value of $g(x)$ when $x=7-w$, where w is a constant. Substituting $7-w$ for x in the given equation yields $g(7-w)=(7-w)(7-w-2)(7-w+6)^2$, which is equivalent to $g(7-w)=(7-w)(5-w)(13-w)^2$. It's given that the value of $g(7-w)$ is 0. Substituting 0 for $g(7-w)$ in the equation $g(7-w)=(7-w)(5-w)(13-w)^2$ yields $0=(7-w)(5-w)(13-w)^2$. Since the product of the three factors on the right-hand side of this equation is equal to 0, at least one of these three factors must be equal to 0. Therefore, the possible values of w can be found by setting each factor equal to 0. Setting the first factor equal to 0 yields $7-w=0$. Adding w to both sides of this equation yields $7=w$. Therefore, 7 is one possible value of w. Setting the second factor equal to 0 yields $5-w=0$. Adding w to both sides of this equation yields $5=w$. Therefore, 5 is a second possible value of w. Setting the third factor equal to 0 yields $(13-w)^2=0$. Taking the square root of both sides of this equation yields $13-w=0$. Adding w to both sides of this equation yields $13=w$. Therefore, 13 is a third possible value of w. Adding the three possible values of w yields $7+5+13$, or 25. Therefore, the sum of all possible values of w is 25.