

1. **Level:** Medium | **Domain:** GEOMETRY AND TRIGONOMETRY
Skill/Knowledge: Circles | **Testing point:** Working with transformations and the equation of a circle

Key Explanation: Choice C is correct. The equation of a circle is given by $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle. In the initial equation, the center of the circle is $(3, -6)$ and in the transformed equation, the center of the circle is $(2, -4)$. To compare the equations, the equation of the second circle must be put into standard form by completing the square as shown below.

$$(x - 2)^2 + y^2 + 8y = 33$$

$$(x - 2)^2 + y^2 + 8y + 16 = 33 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 33 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 49.$$

This yields $(2, -4)$ as the center of the transformed circle.

Therefore, the transformation will be 1 to the left as the x coordinate moves from 3 to 2 and 2 up as the y coordinate moves from -6 to -4 .

Distractor Explanation: Choice A is incorrect because the x coordinate moves to the left rather than to the right. This answer may have been incorrectly selected if the student misinterpreted the standard form for the equation of a circle and thought that the x -coordinates of the centers of the circles are -3 and -2 . Choice B is incorrect. This answer may have been incorrectly selected if the student assumed that the centers were $(-3, 6)$ and $(-2, 4)$ through misinterpretation of the standard form of the equation of a circle and/or through completing the square incorrectly. Choice D is incorrect and may have been incorrectly selected by not completing the square correctly or incorrectly interpreting the center values in the standard equation for a circle

2. **Level:** Hard | **Domain:** ADVANCED MATH
Skill/Knowledge: Nonlinear equations in one variable and systems of equations in two variables
Testing point: How many solutions does an equation have

Key Explanation: Choice B is correct. To solve this equation, first, use the foiling method to expand the left side of the equation and use the distributive property to expand the right side of the equation. Then combine like terms and solve the resulting linear equation as follows:

$$(x - 1)(x + 2) = x(x - 3)$$

$$x^2 - x + 2x - 2 = x^2 - 3x$$

$$x - 2 = -3x$$

$$4x = 2$$

$$x = \frac{1}{2} \text{ or } 0.5$$

This shows that the equation has only one solution.

Distractor Explanation: Choices A and C are incorrect. These answers may be arrived at due to math miscalculations or conceptual errors. Choice D is incorrect. This answer may be arrived at if the student chooses to simply equate each parentheses to zero and finds 1, -2 , 0, and 3 as the solutions.

3. **Level:** Easy | **Domain:** Algebra
Skill/Knowledge: Linear equations in one variable
Testing point: Solving equations of one variable

Key Explanation: For a system of equations to have an infinite number of solutions, it means that the equations are identical and have the same slope and y -intercept.

The first step is to use the distributive property to multiply out the terms on the left side of the equation, resulting in:

$$3x - 18 - 2x - 4 = px - 5x - 22a$$

Combining like terms yields

$$x - 22 = px - 5x - 22$$

Adding $5x$ to both sides yields:

$$6x - 22 = px - 22$$

Therefore p is equal to 6.

4. **Level:** Easy | **Domain:** ADVANCED MATH
Skill/Knowledge: Nonlinear functions | **Testing point:** Determining the meaning in context with exponential equations

Key Explanation: Choice C is correct. The format for an exponential growth equation is:

$$\text{Initial value} \times (1 + \text{growth rate as a decimal})^x$$

The growth factor of the weeds on the farm is defined by $1 +$ the growth rate r .

Therefore,

$$1 + r = 1.995$$

$$r = 0.995 \text{ or } 99.5\%,$$

which would then make choice C correct.

Distractor Explanation: Choice A is incorrect. The number of weeds on the farm, at the beginning, known as the initial value, was 1,300. Choice B is incorrect. This choice implies that the model is linear. However, the model is exponential. Choice D is incorrect because it implies that the model is linear and that 1,300 is the increase in weeds every x weeks, however, it is the initial number of weeds on the farm.

5. **Level:** Easy | **Domain:** ADVANCED MATH
Skill/Knowledge: Equivalent expressions | **Testing point:** Binomial expressions

Key Explanation: Choice A is correct. To solve this problem, first, use the exponent rules to combine both binomials in parentheses with one exponent. $((x - 2)(x + 2))^2$ using the identity $(a + b)(a - b) = a^2 - b^2$, $(x - 2)(x + 2)$ can be simplified to $(x^2 - 4)$ and that would simplify to

$$(x^2 - 4)^2.$$

Using the identity $(a - b)^2 = a^2 - 2ab + b^2$ the problem can be simplified to $(x^2 - 4)^2 = x^4 - 8x^2 + 16$, making Choice A the right option.

Distractor Explanation: Choice B is incorrect. This answer choice may be arrived at due to a common exponent conceptual error. Choice C is incorrect as this answer choice may be due to a miscalculation of $8x$ as positive. Choice D is incorrect due to both exponent conceptual and miscalculation errors.

6. **Level:** Easy | **Domain:** PROBLEM-SOLVING AND DATA ANALYSIS
Skill/Knowledge: One-variable data: distributions and measures of center and spread | **Testing point:** Finding mean and median of grouped data

Key Explanation: To calculate the median of the 27 foods, look for the middle number frequency wise of the data. If there are 27 items then the middle number is the 14th food which, based on the frequency table, has 180 calories. To find the mean number of calories, use the mean formula which can be found by taking the total number of calories divided by the number of food items. To find the total number of calories multiply each calorie content number in the table by its frequency, as follows:

$$(100 \times 6) + (180 \times 9) + (280 \times 11) + (350 \times 1) = \frac{5,650}{27} = 209.3.$$

Therefore the difference of the mean and median of the calorie content of the 27 foods would be $209.3 - 180 = 29.3$.

7. **Level:** Medium | **Domain:** ADVANCED MATH
Skill/Knowledge: Nonlinear functions | **Testing point:** Finding the x -intercept

Key Explanation: Choice C is correct. The x -intercept is found when $y = 0$, therefore:

$$x = 3^{0-1} + \frac{5}{3}$$

$$x = 3^{-1} + \frac{5}{3}$$

$$x = \frac{1}{3} + \frac{5}{3}, \text{ which makes } x = 2 \text{ making Choice C}$$

correct.

Distractor Explanation: Choice A is incorrect. The student might incorrectly think this equation is linear and therefore the $\frac{5}{3}$ represents the y -intercept. However, this is not the case. Also, the question has asked for the x -intercept. Choices B and D are incorrect and would be due to a conceptual or miscalculation error.

8. **Level:** Hard | **Domain:** PROBLEM-SOLVING AND DATA ANALYSIS

Skill/Knowledge: Inferences from sample statistics and margin of error |

Testing point: Generalization of data from surveys

Key Explanation: Choice D is correct. 100% – 80% (who think statistics is the hardest subject) = 20% (who chose Physics as the hardest subject). Since only 11th graders in the math summer class were surveyed the data can only be generalized to the 11th graders in the math summer class. This then makes option D, the best option.

Distractor Explanation: Choice A is incorrect. It generalizes the survey to all students in the summer math class. Choices B and C are incorrect. They contradict the given information.

9. **Level:** Easy | **Domain:** ADVANCED MATH
Skill/Knowledge: Equivalent expressions |
Testing point: Manipulating quadratic equations

Key Explanation: First, expand out $(x + 3)^2$ to $(x + 3) \times (x + 3)$. Using the foiling method, multiply out the two binomial terms to get the following:

$$-3(x^2 + 6x + 9) + 9 + x = ax^2 + bx + c$$

Use the distributive property to expand out the left side of the equation and then combine like terms to get the following: $-3x^2 - 17x - 18 = ax^2 + bx + c$
 Therefore $a = -3$, $b = -17$ and $c = -18$; $a + b + c = (-3) + (-17) + (-18) = -38$

10. **Level:** Hard | **Domain:** ADVANCED MATH
Skill/Knowledge: Nonlinear functions | **Testing point:** Vertex form of a quadratic equation

Key Explanation: Choice B is correct. The maximum point of a quadratic equation is the vertex. Therefore, (3, 2) is the vertex of the parabola. The vertex form of a parabola is given by $a(x - h)^2 + k$, where (h, k) is the vertex of the parabola. This equation can be used to find the equation for a parabola. Therefore, $y = a(x - 3)^2 + 2$. The quadratic equation passes through the point (2, 0). Plugging this point into the vertex form equation for x and y yields

$$0 = a(2 - 3)^2 + 2$$

$$0 = a + 2$$

$$a = -2$$

$$y = -2(x - 3)^2 + 2$$

If x is a solution to the quadratic equation, y must equal zero. Plugging in the answer choices into the

vertex form equation is the fastest method. **Choice B** is the only answer that works. $0 = -2(4 - 3)2 + 2$ and thus $0 = 0$ results.

Distractor Explanation: **Choice A** is incorrect. This is the value of a in the vertex form of a parabola equation. However, this option can also be a miscalculation. **Choice C** is incorrect. This is the y -intercept of the equation. **Choice D** is incorrect. This can be the result of a conceptual or miscalculation error.

11. **Level:** Easy | **Domain:** ALGEBRA

Skill/Knowledge: Linear functions | **Testing point:** Finding the slope of a line given two points

Key Explanation: **Choice C** is correct. To find the

slope of a line, the formula $\frac{y_2 - y_1}{x_2 - x_1}$ is used.

Plugging in the x and y coordinates of the two given points $(0, 0)$ and $(3, 5)$ into the slope

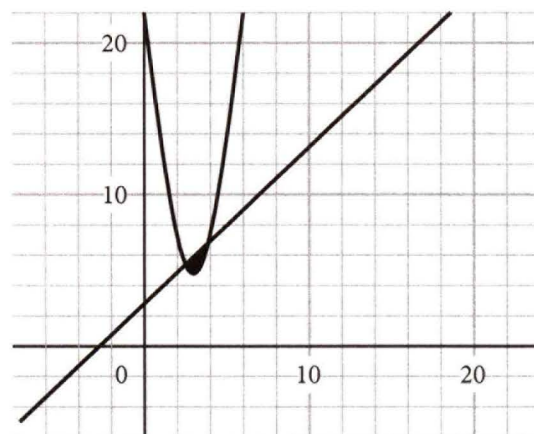
equation yields $\frac{5-0}{3-0} = \frac{5}{3}$

Distractor Explanation: **Choice A** is incorrect and was determined by finding the change in x over the change in y instead of the change in y over the change in x . **Choices B** and **D** are incorrect and were the result of either miscalculation or conceptual errors.

12. **Level:** Hard | **Domain:** ADVANCED MATH

Skill/Knowledge: Nonlinear equations in one variable and systems of equations in two variables
Testing point: System of linear and quadratic inequalities

Key Explanation: **Choice D** is correct. To solve this problem, graph the system.



The solution for the system will be the region that is shaded. The minimum value of b can then be found by finding the vertex of the parabola $y = 2(x - 3)^2 + 5$. The vertex form of a parabola is given by $y = a(x - h)^2 + k$. Therefore, the vertex of the parabola is $(3, 5)$. The minimum y value of a concave up parabola can be found at the vertex. Therefore, the minimum value of b is 5.

Distractor Explanation: **Choice A** is incorrect. It is the minimum value of the system of inequalities. **Choices B** and **C** are incorrect. These answers would be the value of a and b if the vertex was incorrectly found to be $(-3, -5)$.

13. **Level:** Easy | **Domain:** GEOMETRY AND TRIGONOMETRY

Skill/Knowledge: Area and volume | **Testing point:** Volume of a sphere

Key Explanation: The volume of a sphere is found

using the formula $\frac{4}{3}\pi r^3 = \frac{4}{3}(3)^3\pi = 36\pi$

14. **Level:** Easy | **Domain:** ALGEBRA

Skill/Knowledge: Systems of two linear equations in two variables | **Testing point:** Systems of equations with no solution

Key Explanation: Choice A is correct. For a system of equations to have no solution, the system represents parallel lines with the same slope but different y -intercepts.

$$3x - 4y = 8$$

$$px + 8y = -6$$

To easily compare the slopes, the equations should be put into the slope-intercept form of a line $y = mx + b$ where m is the slope. By mathematical manipulation, doing this yields the equations:

$$y = \frac{3}{4}x - 2$$

$$y = \frac{-p}{8}x - \frac{3}{4}$$

Since parallel lines have the same slope, the

equation $\frac{-p}{8} = \frac{3}{4}$ can be solved by p . Multiplying

both sides of the equation by -8 yields $p = -6$,

Choice A.

Distractor Explanation: Choices B, C, and D are incorrect. They would make the system of equations incorrectly have 1 solution. These answer choices are determined by mathematical miscalculations or conceptual errors.

15. **Level:** Easy | **Domain:** GEOMETRY AND TRIGONOMETRY

Skill/Knowledge: Circles | **Testing point:** Equation of a circle

Key Explanation: Choice D is correct. To find the center of the circle, we have to write the equation of the circle in its standard form $(x - h)^2 + (y - k)^2 = r^2$. This is done by completing the square for the

x 's and the y 's in the equation.

$$x^2 + 6x + 9 + y^2 - 6y + 9 = 63 + 9 + 9$$

$$(x + 3)^2 + (y - 3)^2 = 81$$

This then gives the center of the circle as $(-3, 3)$.

Distractor Explanation: Choice A is incorrect.

This is the negative of the center of the equation of the circle.

Choices B and C are incorrect. They can be determined by incorrectly picking the coefficients of x and y without writing the equation in standard form.

16. **Level:** Hard | **Domain:** ADVANCED MATH

Skill/Knowledge: Nonlinear equations in one variable and systems of equations in two variables
Testing point: Solving quadratic equations using the quadratic formula

Key Explanation: This quadratic equation can be solved using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 3 \text{ and } b = -6 \text{ and}$$

$$c = -11.$$

Therefore:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-11)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 + 132}}{6}$$

$$x = \frac{6 \pm \sqrt{168}}{6}$$

$$x = \frac{6 \pm \sqrt{42\sqrt{4}}}{6}$$

$$x = \frac{6 \pm 2\sqrt{42}}{6}$$

$$x = \frac{3 \pm \sqrt{42}}{3}$$

Therefore $g = 42$.

17. **Level:** Easy | **Domain:** ALGEBRA

Skill/Knowledge: Linear inequalities in one or two variables | **Testing point:** Solving linear inequalities

Key Explanation: Choice B is correct. To solve the inequality, add x to both sides of the inequality.

$$-2x + x \geq -4 - x + x$$

$-x \geq -4$, Next, divide both sides of the equation by -1 which flips the inequality sign.

Only 5 is not less than 4.

Distractor Explanation: Choice A is incorrect as it is a solution to the inequality $-4 = -4$.

Choice C is incorrect as it is also a solution to the inequality $3 \leq 4$. Choice D is incorrect as $2 \leq 4$ and is therefore also a solution to the above inequality.

18. **Level:** Easy | **Domain:** ADVANCED MATH

Skill/Knowledge: Nonlinear equations in one variable and systems of equations in two variables

Testing point: Solving square root equations

Key Explanation: Choice B is correct. The problem can be solved in 2 ways. The easiest and quickest way is to just substitute the answer choices and test to see which answer makes the equation true.

Substitute Choice A.

$$\sqrt{2(2)+5} = 2-5$$

$$\sqrt{9} = -3,$$

This isn't true. Substitute Choice B.

$$\sqrt{2(10)+5} = 10-5$$

$$\sqrt{20+5} = 10-5$$

$$\sqrt{25} = 5$$

This makes Choice B the only right option.

Distractor Explanation: Choice A is incorrect mathematically as shown above. Choice C is incorrect as it incorrectly states that 2 is one of the solutions. Choice D is incorrect. It can be due to a miscalculation error.

19. **Level:** Hard | **Domain:** ALGEBRA

Skill/Knowledge: Systems of two linear equations in two variables | **Testing point:** Solving simultaneous equations

Key Explanation: Cross multiplying $\frac{5}{x} = \frac{2}{y}$ yields

$$5y = 2x$$

The next step is to get the two terms on the left side of the second equation of the same common denominator of xy as follows:

$$\frac{4}{x} \times \left(\frac{y}{y}\right) - \frac{2}{y} \times \left(\frac{x}{x}\right) = \frac{4y-2x}{xy} = -\frac{1}{5}$$

Cross multiplying the condensed second equation yields

$$5(4y - 2x) = -xy = 20y - 10x = xy$$

Using the first equation, $5y = 2x$, multiply each side by 4, resulting in $20y = 8x$

Substituting $8x$ for $20y$ in the second equation yields

$$8x - 10x = xy \text{ or } -2x = xy$$

Dividing both sides of the equation by x yields $y = -2$

Substituting this value for y in the first equation yields

$$5(-2) = 2x$$

$$-10 = 2x$$

$$x = -5$$

Therefore, $xy = (-2)(-5) = 10$

20. **Level:** Easy | **Domain:** ADVANCED MATH

Skill/Knowledge: Equivalent expressions |

Testing point: Remainder theorem

Key Explanation: Choice C is correct. A is the remainder when the function $3x^2 + 7x + 9$ is divided by $x - 2$. The remainder can be found when $f(2)$ is calculated. Because $-2 = 0$ yields $x = 2$ as a solution, it can be plugged into the given quadratic function as follows to find A:

$$3(2)^2 + 7(2) + 9 = 12 + 14 + 9 = 35.$$

Distractor Explanation: Choice A is incorrect. This is the answer incorrectly found by adding coefficients together ($3 + 7 + 9$) of the quadratic equation.

Choice B is incorrect. This is the answer incorrectly found by plugging in -2 instead of 2 into the quadratic equation

Choice D is incorrect. This is the answer incorrectly found through a conceptual or miscalculation error.

21. **Level:** Easy | **Domain:** PROBLEM-SOLVING AND DATA ANALYSIS

Skill/Knowledge: Inference from sample statistics and margin of error | **Testing point:** Margin of error

Key Explanation: Choice D is correct. Given that the sample mean mass is 67.9 kgs, the actual mean mass of the penguins in the zoo would lie between 67.9 kgs plus or minus 4.5 kgs, or 63.4 kgs and 72.4 kgs. This then makes Choice D correct.

Distractor Explanation: Choice A is incorrect as the mass of the penguins in the park cannot be found from the data available.

Choice B is incorrect as the mean mass does not state that the mass of all the penguins is 67.9 kgs.

Choice C is incorrect. Nothing can be concluded about the median mass of the penguins in the zoo.

22. **Level:** Hard | **Domain:** ALGEBRA

Skill/Knowledge: Linear equations in two variables | **Testing point:** Number of solutions in a linear system of two variables

Key Explanation: Choice B is correct. The first step is to use the distributive property to multiply out the terms on the left side of the equation as follows:

$$3x - 9 + 2x + 2y = 5x + 2y - 9$$

Combining like terms on the left side of the equation yields

$$5x + 2y - 9 = 5x + 2y - 9$$

Since the equations on the left and the right are the same, they represent the same line and thus have infinitely many solutions.

Distractor Explanation: Choice A is incorrect. To have zero solutions the equations would need to be parallel lines and thus have the same slope and different y-intercepts. That would mean that the coefficients of x and y will be equal however the constants will not. This is not the case in this problem. Choice C is incorrect. That would mean there would be one (x, y) solution to the system above. That is not the case as they represent the same line. Choice D is incorrect. Linear equations with two variables do not have 2 solutions.