Math Test – No Calculator Answer Explanations

Question 1
A babysitter earns $8 an hour for babysitting 2 children and an additional $3 tip when both children are put to bed on time. If the babysitter gets the children to bed on time, what expression could be used to determine how much the babysitter earned?
A) $8x + 3$, where x is the number of hours
B) $3x + 8$, where x is the number of hours
C) $x(8 + 2) + 3$, where x is the number of children
D) $3x + (8 + 2)$, where x is the number of children

Item Difficulty: Easy
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. Let x be the number of hours that the babysitter worked. Since the babysitter earns money at a rate of $8 per hour, she earned $8x$ dollars for the x hours worked. If the babysitter gets both children to bed on time, the babysitter earns an additional $3 tip. Therefore, the babysitter earned a total amount of $8x + 3$ dollars.

Choice B is incorrect since the tip and the rate per hour have been interchanged in the expression. Choices C and D are incorrect since the number of children is not part of how the babysitter's earnings are calculated.

Question 2

$$3(x + y) = y$$

If (x, y) is a solution to the equation above and $y \neq 0$, what is the ratio $\frac{x}{y}$?

A) $-\frac{4}{3}$

B) $-\frac{2}{3}$

C) $\frac{1}{3}$

D) $\frac{2}{3}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: B

Choice B is the correct answer. We can find the ratio $\frac{x}{y}$ by rearranging the equation. Multiplying out the expression on the left side of the equation yields $3x + 3y = y$. Then, subtracting $3y$ from both sides of the equation gives $3x = -2y$. Finally, dividing both sides of this equation by $3y$ (note that $y \neq 0$) gives $\frac{x}{y} = -\frac{2}{3}$

Choices A, C, and D are incorrect; they could result from errors during algebraic transformations of the equation $3(x + y) = y$.

Question 3

$$\frac{1}{2}x - \frac{1}{4}y = 10$$
$$\frac{1}{8}x - \frac{1}{8}y = 19$$

Which ordered pair (x, y) satisfies the system of equations above?

A) $(-112, -264)$

B) $(64, 88)$

C) $\left(\frac{232}{3}, \frac{224}{3}\right)$

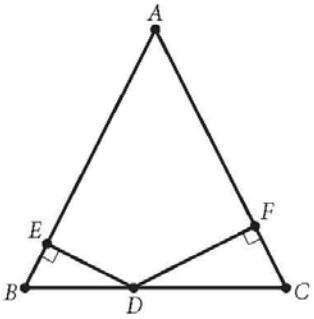
D) $(288, 536)$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. First, we clear the fractions from the two given equations by multiplying both sides of the first equation by 4 and then both sides of the second equation by 8 (note that the new equations are equivalent to the original ones). Thus the system becomes $\begin{cases} 2x - y = 40 \\ x - y = 152 \end{cases}$. Subtracting side by side the second equation from the first eliminates the variable y, $(2x - y) - (x - y) = 40 - 152$, leaving an equation with just one variable, x. Solving this equation gives $x = -112$. Substituting -112 for x into the equation $x - y = 152$ gives $y = -264$. Therefore, $(-112, -264)$ is the ordered pair that satisfies the system of equations given.

Choices B, C, and D are incorrect since the ordered pair in each choice does not satisfy both equations in the system. For example, the ordered pair of choice B, $(64, 88)$, does not satisfy equation $\frac{1}{8}x - \frac{1}{8}y = 19$ because $\frac{1}{8}(64) - \frac{1}{8}(88) \neq 19$.

Question 4



Note: Figure not drawn to scale.

Triangle ABC above is isosceles with $AB = AC$ and $BC = 48$. The ratio of DE to DF is $5 : 7$. What is the length of $\overline{DC}$?

A) 12
B) 20
C) 24
D) 28

Item Difficulty: Medium
Content: Additional Topic in Math
Correct Answer: D

Choice D is the correct answer. The base angles, $\angle B$ and $\angle C$, of isosceles triangle ABC are congruent. Additionally, ΔBED are both right angles and therefore are congruent. Because ΔBED and ΔCFD have two corresponding pairs of angles that are congruent, they are similar. Consequently, the corresponding sides of the similar triangles are proportional. So $\frac{BD}{DC} = \frac{DE}{DF}$, and since $\frac{DE}{DF} = \frac{5}{7}$, it follows that $\frac{BD}{DC} = \frac{5}{7}$. If we let $BD = 5x$, then $DC = 7x$. Since $BD + DC = BC$ and $BC = 48$, it follows that $5x + 7x = 48$. Solving this equation for x gives $x = 4$, and so DC is $7(4) = 28$.

Alternatively: Due to the similarity of ΔBED and ΔCFD, one can conclude that $\frac{BD}{DC} = \frac{5}{7}$, and so DC must be greater than half of BC, which is 24. Of the choices given, only one satisfies this condition, namely 28. If $DC = 28$, then $BD = 48 - 28 = 20$, confirming that $\frac{BD}{DC} = \frac{20}{28} = \frac{5}{7}$. Therefore, the length of $\overline{DC}$ must be 28.

Choices A, B, and C are incorrect because each of the values for DC would result in BC being less than 48 units long.

> Question 5
>
> In a certain game, a player can solve easy or hard puzzles. A player earns 30 points for solving an easy puzzle and 60 points for solving a hard puzzle. Tina solved a total of 50 puzzles playing this game, earning 1,950 points in all. How many hard puzzles did Tina solve?
> A) 10
> B) 15
> C) 25
> D) 35

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: B

Choice B is the correct answer. Let x and y be the number of easy and hard puzzles, respectively, that Tina solved. Since she solved a total of 50 puzzles, it follows that $x + y = 50$. She earned a total of $1{,}950$ points, so it must also be true that $30x + 60y = 1{,}950$. Dividing both sides of this equation by 30 gives $x + 2y = 65$. Subtracting the first equation, $x + y = 50$, from the second equation, $x + 2y = 65$, gives $y = 15$. Therefore, Tina solved 15 hard puzzles.

Alternatively: Let x be the number of easy puzzles Tina solved. Then, $50 - x$ is the number of hard puzzles she solved. And since she earned a total of $1{,}950$ points, it must be true that $30x + 60(50 - x) = 1{,}950$. Solving this equation for x gives $x = 35$, and so $50 - x = 15$. Therefore, Tina solved 15 hard puzzles.

Choices A and C are incorrect because if the number of hard puzzles Tina solved were as they indicate, the total number of points she would earn will not be $1{,}950$. The incorrect answer in choice D could be the result of interchanging the number of hard puzzles and easy puzzles.

Question 6

$$2x^2 + 7x - 15 = 0$$

If r and s are two solutions of the equation above and $r > s$, which of the following is the value of $r - s$?

A) $\frac{15}{2}$

B) $\frac{13}{2}$

C) $\frac{11}{2}$

D) $\frac{3}{2}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: B

Choice B is correct. This equation can be solved using the quadratic formula or factoring. The quadratic formula approach is left as an exercise for students. We will show first how to solve this equation using simple factoring and then will show how to solve it using both the structure of the equation and factoring.

Since $7x = 10x - 3x$, the given equation can be rewritten as $2x^2 + (10x - 3x) - 15 = 0$. Regrouping the terms so that the left side of the equation is in the factored form gives $(2x - 3)(x + 5) = 0$, from which it follows that $2x - 3 = 0$ or $x + 5 = 0$. Thus, the quadratic equation has solutions $\frac{3}{2}$ and -5. Since r and s are solutions to the quadratic equation and $r > s$, we can conclude that $r = \frac{3}{2}$ and $s = -5$; therefore, $r - s = \frac{3}{2} - (-5) = \frac{13}{2}$.

Alternatively: Multiplying the original equation by 2, we can rewrite it in terms of $2x$ as follows: $(2x)^2 + 7(2x) - 30 = 0$. Since the two numbers whose sum is -7 and whose product is -30 are -10 and 3, the equation will be factored as $(2x - 3)(2x + 10) = 0$, generating $\frac{3}{2}$ and -5 as solutions. Since r and s are solutions to the quadratic equation and $r > s$, we can conclude that $r = \frac{3}{2}$ and $s = -5$; therefore, $r - s = \frac{3}{2} - (-5) = \frac{13}{2}$.

Choices A, C, and D are incorrect and could result from calculating the value of expressions given in terms of the solutions r and s, but are not equivalent to the

difference $r - s$ of these solutions. For example, $\frac{15}{2}$ is the value of $-rs$, not the value of $r - s$.

> Question 7
>
> To cut a lawn, Allan charges a fee of $15 for his equipment and $8.50 per hour spent cutting a lawn. Taylor charges a fee of $12 for his equipment and $9.25 per hour spent cutting a lawn. If x represents the number of hours spent cutting a lawn, what are all the values of x for which Taylor's total charge is greater than Allan's total charge?
> A) $x > 4$
> B) $3 \leq x \leq 4$
> C) $4 \leq x \leq 5$
> D) $x < 3$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. If x represents the number of hours spent cutting the lawn, the total fee that Allan charges is $8.5x + 15$ dollars and the total fee that Taylor charges is $9.25x + 12$ dollars. To find all of the values of x for which Taylor's total fee is greater than Allan's total fee, we solve the inequality $9.25x + 12 > 8.5x + 15$, which simplifies to $0.75x > 3$, and so $x > 4$.

Alternatively: Since Taylor's hourly rate charge is higher than Allan's, it can be concluded that after a certain amount of hours, Taylor's total charge will always be greater than Allan's total charge. Thus the inequality that represents all possible values of x for which this occurs will be of the form $x > a$ for some value a. Of the choices given, only $x > 4$ is in this form. Lastly, one can confirm that Taylor and Allan charge the same amount when $x = 4$. Therefore, choice A is correct.

Choice B is incorrect because Allan's total charge is greater than Taylor's total charge when $x < 4$. Choice C is incorrect because Allan's total charge and Taylor's total charge at $x = 4$ are exactly the same, and Taylor's total charge is greater than Allan's total charge also for values of x greater than 5. Choice D is incorrect because Allan's total charge is greater than Taylor's charge when x is less than 3.

Question 8

$$n = 456 - 3T$$

The equation above is used to model the relationship between the number of cups, n, of hot chocolate sold per day in a coffee shop and the average daily temperature, T, in degrees Fahrenheit. According to the model, what is the meaning of the 3 in the equation?

A) For every increase of 3°F, one more cup of hot chocolate will be sold.
B) For every decrease of 3°F, one more cup of hot chocolate will be sold.
C) For every increase of 1°F, three more cups of hot chocolate will be sold.
D) For every decrease of 1°F, three more cups of hot chocolate will be sold.

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: D

Choice D is the correct answer. According to the model, if the average daily temperature is T degrees Fahrenheit, then the number of cups of hot chocolate sold per day in the coffee shop would be $456 - 3T$. If the temperature decreases by 1°F, then the number of cups of hot chocolate sold per day in the coffee shop would be $456 - 3(T - 1)$, which can be rewritten as $(456 - 3T) + 3$. Therefore, for every 1°F drop in the average daily temperature, the coffee shop sells three more cups of hot chocolate.

Choices A and B are incorrect because the change in the average daily temperature and the change in the number of cups of hot chocolate have been interchanged. Choice C is incorrect because, according to the model, the higher value of daily temperature corresponds to a lower, not higher, number of cups of hot chocolate sold.

Question 9

A truck enters a stretch of road that drops 4 meters in elevation for every 100 meters along the length of the road. The road is at 1,300 meters elevation where the truck entered, and the truck is traveling at 16 meters per second along the road. What is the elevation of the road, in meters, at the point where the truck passes t seconds after entering the road?

A) $1{,}300 - 0.04t$

B) $1{,}300 - 0.64t$

C) $1{,}300 - 4t$

D) $1{,}300 - 16t$

Item Difficulty: Medium
Content: Heart of Algebra
Best Answer: B

Choice B is the correct answer. Since the truck is traveling at 16 meters per second along the road, the distance it has traveled t seconds after entering the road is $16t$ meters. Since the elevation of the road drops 4 meters for every 100 meters along the length of the road, it follows that for $16t$ meters along the road, the elevation drops $\frac{4}{100}$ x $16t$ or $0.64t$. Therefore, the elevation of the road at the point where the truck passes t seconds after entering the road is $1{,}300 - 0.64t$ meters.

Choice A is incorrect because $\frac{4}{100}t$ would be the number of meters that the elevation drops t seconds after the truck enters the road if its speed were 1 meter per second. Choice C is incorrect because $4t$ meters does not give the number of meters the elevation of the road drops. Choice D is incorrect because the drop rate of 4 meters for every 100 meters along the road is not used.

Question 10

If $f(x-1)=2x+3$ for all values of x, what is the value of $f(-3)$?

A) -7
B) -5
C) -3
D) -1

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: D

Choice D is correct. Since $f(x-1)=2x+3$ for all values of x, $f(-3)=f(-2-1)=2(-2)+3$ and so the value of $f(-3)$ is -1.

Alternatively: $2x+3$ can be rewritten as $2(x-1)+5$ and since $f(x-1)=2(x-1)+5$ for all values of x, it follows that $f(x)=2x+5$ for all values of x. Substituting -3 for x in this equation gives $f(-3)=2(-3)+5=-1$.

Choices A, B, and C are incorrect because f is a function, and there is one and only one value for $f(-3)$, which as shown above is -1. Therefore, neither of the choices, -7, -5, or -4 can be the value of $f(-3)$.

Question 11

Which of the following is equivalent to $(s-t)\left(\frac{s}{t}\right)$?

A) $\frac{s}{t}-s$
B) $\frac{s}{t}-st$
C) $\frac{s^2}{t}-s$
D) $\frac{s^2}{t}-\frac{s}{t^2}$

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: C

Choice C is the correct answer. Using the distributive property to expand the given expression gives $s\left(\frac{s}{t}\right) - t\left(\frac{s}{t}\right) = \frac{s^2}{t} - s$.

Choices A, B, and D are incorrect. In each of these choices, at least one of the products in the expansion is not correct. For example $s\left(\frac{s}{t}\right) = \frac{s^2}{t}$, not $\frac{s}{t}$, and $t\left(\frac{s}{t}\right) = s$, not st or $\frac{s}{t}$.

Question 12

$$p(x) = 3(x^2 + 10x + 5) - 5(x - k)$$

In the polynomial $p(x)$ defined above, k is a constant. If $p(x)$ is divisible by x, what is the value of k?

A) −3
B) −2
C) 0
D) 3

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: A

Choice A is the correct answer. If polynomial $p(x)$ is divisible by x, then x must be a factor of the polynomial, or equivalently, the constant term of the polynomial must be zero. Multiplying out on the right side of the equation gives $p(x) = 3x^2 + 30x + 15 - 5x + 5k$, which can be rewritten as $p(x) = 3x^2 + 25x + (5k + 15)$. Hence, $5k + 15 = 0$, and so $k = -3$.

Choices B, C, and D are the not correct answers because if the value of k were as indicated in those choices, then x would not be a factor of the polynomial $p(x)$, and so $p(x)$ would not be divisible by x.

Question 13

In the xy-plane, if the parabola with equation $y = ax2 + bx + c$, where a, b, and c are constants, passes through the point $(-1, 1)$, which of the following must be true?

A) $a - b = 1$
B) $-b + c = 1$
C) $a + b + c = 1$
D) $a - b + c = 1$

Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: D

Choice D is the correct answer. If the graph of a parabola passes through the point $(-1, 1)$, then the ordered pair $(-1, 1)$ must satisfy the equation of the parabola. Thus, $1 = a(-1)^2 + b(-1) + c$, which is equivalent to $a - b + c = 1$.

Choices A, B, and C are incorrect and could result from misinterpreting what it means for the point (–1, 1) to be on the parabola or from common calculation errors while expressing this fact algebraically.
These are the directions students will see in the test for the Student-Produced Response questions.

Question 14

For what value of h is $24 = \frac{h}{10} - 6$?

Item Difficulty: Easy
Content: Heart of Algebra
Correct Answer: 300

The correct answer is 300. To solve the given equation for h, first add 6 to both sides of the equation to get $30 = \frac{h}{10}$. Then multiply both sides of this equation by 10 to yield $h = 300$.

> Question 15
>
> What is the value of a if $(2a + 3) - (4a - 8) = 7$?

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: 2

The correct answer is 2. The equation given can be rewritten as $2a + 3 - 4a + 8 = 7$, which is equivalent to $-2a + 11 = 7$, and so $a = 2$.

> Question 16
>
> If x is not equal to zero, what is the value of $\frac{4(3x)^2}{(2x)^2}$?

Item Difficulty: Medium
Content: Passport to Advanced Math
Correct Answer: 9

The correct answer is 9. Multiplying out the given expression gives $\frac{4(9x^2)}{4x^2}$. Since $x \neq 0$, dividing both the numerator and the denominator of the fraction by $4x^2$ simplifies the expression to 9.

> Question 17
>
> If $x - 2$ is a factor of $x^2 - bx + b$, where b is a constant, what is the value of b ?

Item Difficulty: Hard
Content: Passport to Advanced Math
Correct Answer: 4

The correct answer is 4. If $x - 2$ is a factor of $x^2 - bx + b$, where b is a constant, then $x^2 - bx + b$ can be written as the product $(x - 2)(x - a)$ for some real number a. Expanding $(x - 2)(x - a)$ gives $x^2 - 2x - ax + 2a$, which can be rewritten as $x^2 - (2 + a)x + 2a$. Hence, $x^2 - (2 + a)x + 2a = x^2 - bx + b$ is true for all values of x. Consequently, the coefficients of like terms on each side of the equation must be the same: $2 + a = b$ and $2a = b$. Solving this system gives $b = 4$.

Alternatively: Since $x - 2$ is a factor of $x^2 - bx + b$ and $(x - 2)^2 = x^2 - 4x + 4$,one can correctly conclude that the value of b is 4.