

Question ID 8d3089b0

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 8d3089b0

Bacteria are growing in a liquid growth medium. There were **300,000** cells per milliliter during an initial observation. The number of cells per milliliter doubles every **3** hours. How many cells per milliliter will there be **15** hours after the initial observation?

- A. 1,500,000
- B. 2,400,000
- C. 4,500,000
- D. 9,600,000

ID: 8d3089b0 Answer

Correct Answer: D

Rationale

Choice D is correct. Let y represent the number of cells per milliliter x hours after the initial observation. Since the number of cells per milliliter doubles every 3 hours, the relationship between x and y can be represented by an exponential equation of the form $y = ab^{\frac{x}{k}}$, where a is the number of cells per milliliter during the initial observation and the number of cells per milliliter increases by a factor of b every k hours. It's given that there were 300,000 cells per milliliter during the initial observation. Therefore, $a = 300,000$. It's also given that the number of cells per milliliter doubles, or increases by a factor of 2, every 3 hours. Therefore, $b = 2$ and $k = 3$. Substituting 300,000 for a , 2 for b , and 3 for k in the equation $y = ab^{\frac{x}{k}}$ yields $y = 300,0002^{\frac{x}{3}}$. The number of cells per milliliter there will be 15 hours after the initial observation is the value of y in this equation when $x = 15$. Substituting 15 for x in the equation $y = 300,0002^{\frac{x}{3}}$ yields $y = 300,0002^{\frac{15}{3}}$, or $y = 300,0002^5$. This is equivalent to $y = 300,00032$, or $y = 9,600,000$. Therefore, 15 hours after the initial observation, there will be 9,600,000 cells per milliliter.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 89160f9c

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 89160f9c

$f(x) = x^2 - 48x + 2,304$

What is the minimum value of the given function?

ID: 89160f9c Answer

Correct Answer: 1728

Rationale

The correct answer is 1,728. The given function can be rewritten in the form $fx = ax - h^2 + k$, where a is a positive constant and the minimum value, k , of the function occurs when the value of x is h . By completing the square, $fx = x^2 - 48x + 2,304$ can be written as $fx = x^2 - 48x + \frac{48^2}{2} + 2,304 - \frac{48^2}{2}$, or $fx = x - 24^2 + 1,728$. This equation is in the form $fx = ax - h^2 + k$, where $a = 1$, $h = 24$, and $k = 1,728$. Therefore, the minimum value of the given function is 1,728.

Question Difficulty: Hard

Question ID 9e74d905

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 9e74d905

When the quadratic function f is graphed in the xy -plane, where $y = f(x)$, its vertex is $(-3, 6)$. One of the x -intercepts of this graph is $(-\frac{17}{4}, 0)$. What is the other x -intercept of the graph?

- A. $(-\frac{29}{4}, 0)$
- B. $(-\frac{7}{4}, 0)$
- C. $(\frac{5}{4}, 0)$
- D. $(\frac{17}{4}, 0)$

ID: 9e74d905 Answer

Correct Answer: B

Rationale

Choice B is correct. Since the line of symmetry for the graph of a quadratic function contains the vertex of the graph, the x -coordinate of the vertex of the graph of $y = f(x)$ is the x -coordinate of the midpoint of its two x -intercepts. The midpoint of two points with x -coordinates x_1 and x_2 has x -coordinate x_m , where $x_m = \frac{x_1 + x_2}{2}$. It's given that the vertex is $-3, 6$ and one of the x -intercepts is $-\frac{17}{4}, 0$. Substituting -3 for x_m and $-\frac{17}{4}$ for x_1 in the equation $x_m = \frac{x_1 + x_2}{2}$ yields $-3 = \frac{-\frac{17}{4} + x_2}{2}$. Multiplying each side of this equation by 2 yields $-6 = -\frac{17}{4} + x_2$. Adding $\frac{17}{4}$ to each side of this equation yields $-\frac{7}{4} = x_2$. Therefore, the other x -intercept is $-\frac{7}{4}, 0$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d1b3d70e

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: d1b3d70e

Square P has a side length of x inches. Square Q has a perimeter that is **176** inches greater than the perimeter of square P. The function f gives the area of square Q, in square inches. Which of the following defines f ?

- A. $f(x) = (x + 44)^2$
- B. $f(x) = (x + 176)^2$
- C. $f(x) = (176x + 44)^2$
- D. $f(x) = (176x + 176)^2$

ID: d1b3d70e Answer

Correct Answer: A

Rationale

Choice A is correct. Let x represent the side length, in inches, of square P. It follows that the perimeter of square P is $4x$ inches. It's given that square Q has a perimeter that is 176 inches greater than the perimeter of square P. Thus, the perimeter of square Q is 176 inches greater than $4x$ inches, or $4x + 176$ inches. Since the perimeter of a square is 4 times the side length of the square, each side length of Q is $\frac{4x + 176}{4}$, or $x + 44$ inches. Since the area of a square is calculated by multiplying the length of two sides, the area of square Q is $x + 44x + 44$, or $x + 44^2$ square inches. It follows that function f is defined by $fx = x + 44^2$.

Choice B is incorrect. This function represents a square with side lengths $x + 176$ inches.

Choice C is incorrect. This function represents a square with side lengths $176x + 44$ inches.

Choice D is incorrect. This function represents a square with side lengths $176x + 176$ inches.

Question Difficulty: Hard

Question ID b11955c5

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: b11955c5

The functions f and g are defined by the given equations.

$$f(x) = 3 + \left| -2x - x^2 \right|$$

$$g(w) = \left| \frac{-w}{w-1} \right| - w + 5$$

If $f(-4) = c$, where c is a constant, what is the value of $g(c)$?

ID: b11955c5 Answer

Correct Answer: -4.9, -49/10

Rationale

The correct answer is -4.9. The value of $f-4$ is the value of fx when $x = -4$. Substituting -4 for x in the equation $fx = 3 + -2x - x^2$ yields $f-4 = 3 + -2-4 - -4^2$, or $f-4 = 3 + -8$, which is equivalent to $f-4 = 3 + 8$, or $f-4 = 11$. Since it's given that $f-4 = c$, it follows that $c = 11$ and the value of gc is the value of $g11$. Substituting 11 for w in the equation $gw = \frac{-w}{w-1} - w + 5$ yields $g11 = \frac{-11}{11-1} - 11 + 5$, or $g11 = -1.1 - 6$, which is equivalent to $g11 = 1.1 - 6$, or $g11 = -4.9$. Note that -4.9 and -49/10 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID b2142aba

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: b2142aba

For the exponential function f , the value of $f(1)$ is k , where k is a constant. Which of the following equivalent forms of the function f shows the value of k as the coefficient or the base?

- A. $f(x) = 50(2)^{x+1}$
- B. $f(x) = 80(2)^x$
- C. $f(x) = 128(2)^{x-1}$
- D. $f(x) = 205(2)^{x-2}$

ID: b2142aba Answer

Correct Answer: C

Rationale

Choice C is correct. For the form of the function in choice C, $fx = 1281.6^{x-1}$, the value of $f1$ can be found as 1281.6^{1-1} , which is equivalent to 1281.6^0 , or 128. Therefore, $k = 128$, which is shown in $fx = 1281.6^{x-1}$ as the coefficient.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 6b7c82dd

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 6b7c82dd

The function $f(x) = \frac{1}{9}(x - 7)^2 + 3$ gives a metal ball's height above the ground $f(x)$, in inches, x seconds after it started moving on a track, where $0 \leq x \leq 10$. Which of the following is the best interpretation of the vertex of the graph of $y = f(x)$ in the xy -plane?

- A. The metal ball's minimum height was 3 inches above the ground.
- B. The metal ball's minimum height was 7 inches above the ground.
- C. The metal ball's height was 3 inches above the ground when it started moving.
- D. The metal ball's height was 7 inches above the ground when it started moving.

ID: 6b7c82dd Answer

Correct Answer: A

Rationale

Choice A is correct. The graph of a quadratic equation in the form $y = ax - h^2 + k$, where a , h , and k are positive constants, is a parabola that opens upward with vertex h, k . The given function $fx = \frac{1}{9}x - 7^2 + 3$ is in the form $y = ax - h^2 + k$, where $y = fx$, $a = \frac{1}{9}$, $h = 7$, and $k = 3$. Therefore, the graph of $y = fx$ is a parabola that opens upward with vertex 7, 3. Since the parabola opens upward, the vertex is the lowest point on the graph. It follows that the y -coordinate of the vertex of the graph of $y = fx$ is the minimum value of fx . Therefore, the minimum value of fx is 3. It's given that $fx = \frac{1}{9}x - 7^2 + 3$ represents the metal ball's height above the ground, in inches, x seconds after it started moving on a track. Therefore, the best interpretation of the vertex of the graph of $y = fx$ is that the metal ball's minimum height was 3 inches above the ground.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 71875bb0

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 71875bb0

For the function q , the value of $q(x)$ decreases by 45% for every increase in the value of x by 1. If $q(0) = 14$, which equation defines q ?

- A. $q(x) = 0.55(14)^x$
- B. $q(x) = 1.45(14)^x$
- C. $q(x) = 14(0.55)^x$
- D. $q(x) = 14(1.45)^x$

ID: 71875bb0 Answer

Correct Answer: C

Rationale

Choice C is correct. Since the value of qx decreases by a fixed percentage, 45%, for every increase in the value of x by 1, the function q is a decreasing exponential function. A decreasing exponential function can be written in the form $qx = a1 - \frac{p}{100}^x$, where a is the value of $q0$ and the value of qx decreases by $p\%$ for every increase in the value of x by 1. If $q0 = 14$, then $a = 14$. Since the value of qx decreases by 45% for every increase in the value of x by 1, $p = 45$. Substituting 14 for a and 45 for p in the equation $qx = a1 - \frac{p}{100}^x$ yields $qx = 141 - \frac{45}{100}^x$, which is equivalent to $qx = 141 - 0.45^x$, or $qx = 140.55^x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

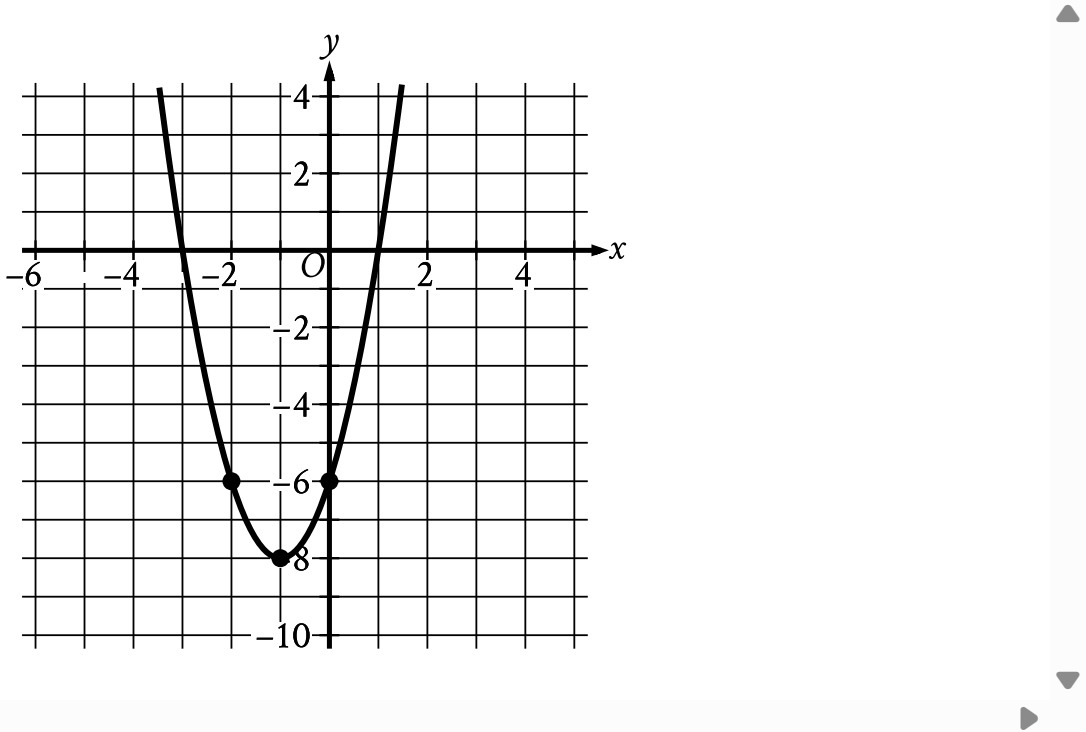
Choice D is incorrect. For this function, the value of qx increases, rather than decreases, by 45% for every increase in the value of x by 1.

Question Difficulty: Hard

Question ID d1cf915b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: d1cf915b



The graph of $y = 2x^2 + bx + c$ is shown, where b and c are constants. What is the value of bc ?

ID: d1cf915b Answer

Correct Answer: -24

Rationale

The correct answer is -24. Since the graph passes through the point 0, - 6, it follows that when the value of x is 0, the value of y is -6. Substituting 0 for x and -6 for y in the given equation yields $-6 = 2(0)^2 + b(0) + c$, or $-6 = c$. Therefore, the value of c is -6. Substituting -6 for c in the given equation yields $y = 2x^2 + bx - 6$. Since the graph passes through the point -1, - 8, it follows that when the value of x is -1, the value of y is -8. Substituting -1 for x and -8 for y in the equation $y = 2x^2 + bx - 6$ yields $-8 = 2(-1)^2 + b(-1) - 6$, or $-8 = 2 - b - 6$, which is equivalent to $-8 = -4 - b$. Adding 4 to each side of this equation yields $-4 = -b$. Dividing each side of this equation by -1 yields $4 = b$. Since the value of b is 4 and the value of c is -6, it follows that the value of bc is $4(-6)$, or -24.

Alternate approach: The given equation represents a parabola in the xy -plane with a vertex at -1, - 8. Therefore, the given equation, $y = 2x^2 + bx + c$, which is written in standard form, can be written in vertex form, $y = a(x - h)^2 + k$, where h, k is the vertex of the parabola and a is the value of the coefficient on the x^2 term when the equation is written in standard form. It follows that $a = 2$. Substituting 2 for a , -1 for h , and -8 for k in this equation yields $y = 2(x - (-1))^2 - 8$, or $y = 2(x + 1)^2 - 8$. Squaring the binomial on the right-hand side of this equation yields $y = 2(x^2 + 2x + 1) - 8$. Multiplying each term inside the parentheses on the right-hand side of this equation by 2 yields $y = 2x^2 + 4x + 2 - 8$, which is equivalent to $y = 2x^2 + 4x - 6$. From the given equation $y = 2x^2 + bx + c$, it follows that the value of b is 4 and the value of c is -6. Therefore, the value of bc is $4(-6)$, or -24.

Question ID af172557

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: af172557

$f(x) = 9(4)^x$

The function f is defined by the given equation. If $g(x) = f(x + 2)$, which of the following equations defines the function g ?

- A. $g(x) = 18(4)^x$
- B. $g(x) = 144(4)^x$
- C. $g(x) = 18(8)^x$
- D. $g(x) = 81(16)^x$

ID: af172557 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $fx = 94^x$ and $gx = fx + 2$. Substituting $x + 2$ for x in $fx = 94^x$ gives $fx + 2 = 94^{x + 2}$. Rewriting this equation using properties of exponents gives $fx + 2 = 94^x 4^2$, which is equivalent to $fx + 2 = 94^x 16$. Multiplying 9 and 16 in this equation gives $fx + 2 = 1444^x$. Since $gx = fx + 2$, $gx = 1444^x$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ec792518

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: ec792518

$f(x) = |59 - 2x|$

The function f is defined by the given equation. For which of the following values of k does $f(k) = 3k$?

- A. $\frac{59}{5}$
- B. $\frac{59}{2}$
- C. $\frac{177}{5}$
- D. 59

ID: ec792518 Answer

Correct Answer: A

Rationale

Choice A is correct. The value of k for which $fk = 3k$ can be found by substituting k for x and $3k$ for fx in the given equation, $fx = 59 - 2x$, which yields $3k = 59 - 2k$. For this equation to be true, either $-3k = 59 - 2k$ or $3k = 59 - 2k$. Adding $2k$ to both sides of the equation $-3k = 59 - 2k$ yields $-k = 59$. Dividing both sides of this equation by -1 yields $k = -59$. To check whether -59 is the value of k , substituting -59 for k in the equation $3k = 59 - 2k$ yields $3(-59) = 59 - 2(-59)$, which is equivalent to $-177 = 177$, or $-177 = 177$, which isn't a true statement. Therefore, -59 isn't the value of k . Adding $2k$ to both sides of the equation $3k = 59 - 2k$ yields $5k = 59$. Dividing both sides of this equation by 5 yields $k = \frac{59}{5}$. To check whether $\frac{59}{5}$ is the value of k , substituting $\frac{59}{5}$ for k in the equation $3k = 59 - 2k$ yields $3(\frac{59}{5}) = 59 - 2(\frac{59}{5})$, which is equivalent to $\frac{177}{5} = \frac{177}{5}$, or $\frac{177}{5} = \frac{177}{5}$, which is a true statement. Therefore, the value of k for which $fk = 3k$ is $\frac{59}{5}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d4b3c7eb

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: d4b3c7eb

$f(x) = 272(2)^x$

The function f is defined by the given equation. If $h(x) = f(x - 4)$, which of the following equations defines function h ?

- A. $h(x) = 17(2)^x$
- B. $h(x) = 68(2)^x$
- C. $h(x) = 272(16)^x$
- D. $h(x) = 272(8)^x$

ID: d4b3c7eb Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $f(x) = 272(2)^x$ and $h(x) = f(x - 4)$. Substituting $x - 4$ for x in $f(x) = 272(2)^x$ yields $f(x - 4) = 272(2)^{x - 4}$. Substituting hx for $f(x - 4)$ in this equation yields $hx = 272(2)^{x - 4}$. Using the properties of exponents, the function $h(x) = 272(2)^{x - 4}$ can be rewritten as $hx = \frac{2722^x}{2^4}$, which is equivalent to $hx = \frac{2722^x}{16}$, or $h(x) = 17(2)^x$. Therefore, of the given choices, an equation that defines function h is $h(x) = 17(2)^x$.

Choice B is incorrect. This equation defines function h if $h(x) = f(x - 2)$, not $h(x) = f(x - 4)$.

Choice C is incorrect. This equation defines function h if $h(x) = f(4x)$, not $h(x) = f(x - 4)$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID de3eaf05

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: de3eaf05

$$P(t) = 290(1.04)^{\left(\frac{4}{6}\right)t}$$

The function P models the population, in thousands, of a certain city t years after 2005. According to the model, the population is predicted to increase by $n\%$ every 18 months. What is the value of n ?

- A. 0.38
- B. 1.04
- C. 4
- D. 6

ID: de3eaf05 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function P models the population of the city t years after 2005. Since there are 12 months in a year, 18 months is equivalent to $\frac{18}{12}$ years. Therefore, the expression $\frac{18}{12}x$ can represent the number of years in x 18-month periods. Substituting $\frac{18}{12}x$ for t in the given equation yields $P\frac{18}{12}x = 2901.04^{\frac{4}{6}\frac{18}{12}x}$, which is equivalent to $P\frac{18}{12}x = 2901.04^x$. Therefore, for each 18-month period, the predicted population of the city is 1.04 times, or 104% of, the previous population. This means that the population is predicted to increase by 4% every 18 months.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. Each year, the predicted population of the city is 1.04 times the previous year's predicted population, which is not the same as an increase of 1.04%.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 067262c7

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 067262c7

The function f is defined by $f(x) = ax^2 + bx + c$, where a , b , and c are constants. The graph of $y = f(x)$ in the xy -plane passes through the points $(7, 0)$ and $(-3, 0)$. If a is an integer greater than 1, which of the following could be the value of $a + b$?

- A. -6
- B. -3
- C. 4
- D. 5

ID: 067262c7 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graph of $y = fx$ in the xy -plane passes through the points 7, 0 and -3, 0. It follows that when the value of x is either 7 or -3, the value of fx is 0. It's also given that the function f is defined by $fx = ax^2 + bx + c$, where a , b , and c are constants. It follows that the function f is a quadratic function and, therefore, may be written in factored form as $fx = ax - ux - v$, where the value of fx is 0 when x is either u or v . Since the value of fx is 0 when the value of x is either 7 or -3, and the value of fx is 0 when the value of x is either u or v , it follows that u and v are equal to 7 and -3. Substituting 7 for u and -3 for v in the equation $fx = ax - ux - v$ yields $fx = ax - 7x - -3$, or $fx = ax - 7x + 3$. Distributing the right-hand side of this equation yields $fx = ax^2 - 7x + 3x - 21$, or $fx = ax^2 - 4ax - 21a$. Since it's given that $fx = ax^2 + bx + c$, it follows that $b = -4a$. Adding a to each side of this equation yields $a + b = -3a$. Since $a + b = -3a$, if a is an integer, the value of $a + b$ must be a multiple of 3. If a is an integer greater than 1, it follows that $a \geq 2$. Therefore, $-3a \leq -32$. It follows that the value of $a + b$ is less than or equal to -32, or -6. Of the given choices, only -6 is a multiple of 3 that's less than or equal to -6.

Choice B is incorrect. This is the value of $a + b$ if a is equal to, not greater than, 1.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 3d0f48a9

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 3d0f48a9

$$P(t) = 260(1.04)^{(\frac{6}{4})t}$$

The function P models the population, in thousands, of a certain city t years after **2003**. According to the model, the population is predicted to increase by **4%** every n months. What is the value of n ?

- A. **8**
- B. **12**
- C. **18**
- D. **72**

ID: 3d0f48a9 Answer

Correct Answer: A

Rationale

Choice A is correct. It’s given that the function P models the population, in thousands, of a certain city t years after 2003. The value of the base of the given exponential function, 1.04, corresponds to an increase of 4% for every increase of 1 in the exponent, $\frac{6}{4}t$. If the exponent is equal to 0, then $\frac{6}{4}t = 0$. Multiplying both sides of this equation by $\frac{4}{6}$ yields $t = 0$. If the exponent is equal to 1, then $\frac{6}{4}t = 1$. Multiplying both sides of this equation by $\frac{4}{6}$ yields $t = \frac{4}{6}$, or $t = \frac{2}{3}$. Therefore, the population is predicted to increase by 4% every $\frac{2}{3}$ of a year. It’s given that the population is predicted to increase by 4% every n months. Since there are 12 months in a year, $\frac{2}{3}$ of a year is equivalent to $\frac{2}{3}12$, or 8, months. Therefore, the value of n is 8.

Choice B is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $Pt = 2601.04^t$, not $Pt = 2601.04^{\frac{6}{4}t}$.

Choice C is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $Pt = 2601.04^{\frac{4}{6}t}$, not $Pt = 2601.04^{\frac{6}{4}t}$.

Choice D is incorrect. This is the number of months in which the population is predicted to increase by 4% according to the model $Pt = 2601.04^{\frac{1}{6}t}$, not $Pt = 2601.04^{\frac{6}{4}t}$.

Question Difficulty: Hard

Question ID 8e87347e

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 8e87347e

$$f(x) = 9,000(0.66)^x$$

The given function f models the number of advertisements a company sent to its clients each year, where x represents the number of years since 1997, and $0 \leq x \leq 5$. If $y = f(x)$ is graphed in the xy -plane, which of the following is the best interpretation of the y -intercept of the graph in this context?

- A. The minimum estimated number of advertisements the company sent to its clients during the 5 years was 1,708.
- B. The minimum estimated number of advertisements the company sent to its clients during the 5 years was 9,000.
- C. The estimated number of advertisements the company sent to its clients in 1997 was 1,708.
- D. The estimated number of advertisements the company sent to its clients in 1997 was 9,000.

ID: 8e87347e Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of a graph in the xy -plane is the point where $x = 0$. For the given function f , the y -intercept of the graph of $y = fx$ in the xy -plane can be found by substituting 0 for x in the equation $y = 9,000(0.66)^x$, which gives $y = 9,000(0.66)^0$. This is equivalent to $y = 9,000(1)$, or $y = 9,000$. Therefore, the y -intercept of the graph of $y = fx$ is 9,000. It's given that the function f models the number of advertisements a company sent to its clients each year. Therefore, fx represents the estimated number of advertisements the company sent to its clients each year. It's also given that x represents the number of years since 1997. Therefore, $x = 0$ represents 0 years since 1997, or 1997. Thus, the best interpretation of the y -intercept of the graph of $y = fx$ is that the estimated number of advertisements the company sent to its clients in 1997 was 9,000.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a25d39ec

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: a25d39ec

The product of two positive integers is **462**. If the first integer is **5** greater than twice the second integer, what is the smaller of the two integers?

ID: a25d39ec Answer

Correct Answer: 14

Rationale

The correct answer is 14. Let x represent the first integer and y represent the second integer. If the first integer is 5 greater than twice the second integer, then $x = 2y + 5$. It's given that the product of the two integers is 462; therefore $xy = 462$. Substituting $2y + 5$ for x in this equation yields $(2y + 5)(y) = 462$, which can be written as $2y^2 + 5y = 462$. Subtracting 462 from each side of this equation yields $2y^2 + 5y - 462 = 0$. The left-hand side of this equation can be factored by finding two values whose product is $2(-462)$, or -924, and whose sum is 5. The two values whose product is -924 and whose sum is 5 are 33 and -28. Thus, the equation $2y^2 + 5y - 462 = 0$ can be rewritten as $2y^2 - 28y + 33y - 462 = 0$, which is equivalent to $2y(y - 14) + 33(y - 14) = 0$, or $(2y + 33)(y - 14) = 0$. By the zero product property, it follows that $2y + 33 = 0$ or $y - 14 = 0$. Subtracting 33 from both sides of the equation $2y + 33 = 0$ yields $2y = -33$. Dividing both sides of this equation by 2 yields $y = -\frac{33}{2}$. Since y is a positive integer, the value of y isn't $-\frac{33}{2}$. Adding 14 to both sides of the equation $y - 14 = 0$ yields $y = 14$. Substituting 14 for y in the equation $xy = 462$ yields $x(14) = 462$. Dividing both sides of this equation by 14 yields $x = 33$. Therefore, the two integers are 14 and 33, so the smaller of the two integers is 14.

Question Difficulty: Hard

Question ID e89e2a0f

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: e89e2a0f

The functions f and g are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$?

I. $f(x) = 33(0.4)^{x+3}$

II. $g(x) = 33(0.16)(0.4)^{x-2}$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: e89e2a0f Answer

Correct Answer: B

Rationale

Choice B is correct. Functions f and g are both exponential functions with a base of 0.40. Since 0.40 is less than 1, functions f and g are both decreasing exponential functions. This means that fx and gx decrease as x increases. Since fx and gx decrease as x increases, the maximum value of each function occurs at the least value of x for which the function is defined. It's given that functions f and g are defined for $x \geq 0$. Therefore, the maximum value of each function occurs at $x = 0$. Substituting 0 for x in the equation defining f yields $f0 = 330.4^{0+3}$, which is equivalent to $f0 = 330.4^3$, or $f0 = 2.112$. Therefore, the maximum value of f is 2.112. Since the equation $fx = 330.4^{x+3}$ doesn't display the value 2.112, the equation defining f doesn't display the maximum value of f . Substituting 0 for x in the equation defining g yields $g0 = 330.160.4^{0-2}$, which can be rewritten as $g0 = 330.16\frac{1}{0.4^2}$, or $g0 = 330.16\frac{1}{0.16}$, which is equivalent to $g0 = 33$. Therefore, the maximum value of g is 33. Since the equation $gx = 330.160.4^{x-2}$ displays the value 33, the equation defining g displays the maximum value of g . Thus, only equation II displays, as a constant or coefficient, the maximum value of the function it defines.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2e2b270a

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 2e2b270a

$f(x) = (1.84)^{\frac{x}{4}}$

The function f is defined by the given equation. The equation can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant. Which of the following is closest to the value of p ?

- A. 16
- B. 21
- C. 46
- D. 96

ID: 2e2b270a Answer

Correct Answer: A

Rationale

Choice A is correct. The equation $fx = 1.84^{\frac{x}{4}}$ can be rewritten as $fx = 1.84^{\frac{1}{4}x}$, which is equivalent to $fx = 1.84^{\frac{1}{4}x}$, or approximately $fx = 1.16467^x$. Since it's given that $fx = 1.84^{\frac{x}{4}}$ can be rewritten as $fx = 1 + \frac{p}{100}^x$, where p is a constant, it follows that $1 + \frac{p}{100}$ is approximately equal to 1.16467. Therefore, $\frac{p}{100}$ is approximately equal to 0.16467. It follows that the value of p is approximately equal to 16.467. Of the given choices, 16 is closest to the value of p .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 92fecab6

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 92fecab6

The function $f(x) = 206(1.034)^x$ models the value, in dollars, of a certain bank account by the end of each year from 1957 through 1972, where x is the number of years after 1957. Which of the following is the best interpretation of “ $f(5)$ is approximately equal to 243” in this context?

- A. The value of the bank account is estimated to be approximately 5 dollars greater in 1962 than in 1957.
- B. The value of the bank account is estimated to be approximately 243 dollars in 1962.
- C. The value, in dollars, of the bank account is estimated to be approximately 5 times greater in 1962 than in 1957.
- D. The value of the bank account is estimated to increase by approximately 243 dollars every 5 years between 1957 and 1972.

ID: 92fecab6 Answer

Correct Answer: B

Rationale

Choice B is correct. It’s given that the function $f(x) = 206(1.034)^x$ models the value, in dollars, of a certain bank account by the end of each year from 1957 through 1972, where x is the number of years after 1957. It follows that $f(x)$ represents the estimated value, in dollars, of the bank account x years after 1957. Since the value of $f(5)$ is the value of $f(x)$ when $x = 5$, it follows that “ $f(5)$ is approximately equal to 243” means that $f(x)$ is approximately equal to 243 when $x = 5$. In the given context, this means that the value of the bank account is estimated to be approximately 243 dollars 5 years after 1957. Therefore, the best interpretation of the statement “ $f(5)$ is approximately equal to 243” in this context is the value of the bank account is estimated to be approximately 243 dollars in 1962.

Choice A is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID e3b52d28

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: e3b52d28

Two variables, x and y , are related such that for each increase of 1 in the value of x , the value of y increases by a factor of 4. When $x = 0$, $y = 200$. Which equation represents this relationship?

- A. $y = 4(x)^{200}$
- B. $y = 4(200)^x$
- C. $y = 200(x)^4$
- D. $y = 200(4)^x$

ID: e3b52d28 Answer

Correct Answer: D

Rationale

Choice D is correct. Since the value of y increases by a constant factor, 4, for each increase of 1 in the value of x , the relationship between x and y is exponential. An exponential relationship between x and y can be represented by an equation of the form $y = ab^x$, where a is the value of x when $y = 0$ and y increases by a factor of b for each increase of 1 in the value of x . Since $y = 200$ when $x = 0$, $a = 200$. Since y increases by a factor of 4 for each increase of 1 in the value of x , $b = 4$. Substituting 200 for a and 4 for b in the equation $y = ab^x$ yields $y = 2004^x$. Thus, the equation $y = 2004^x$ represents the relationship between x and y .

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect. This equation represents a relationship where for each increase of 1 in the value of x , the value of y increases by a factor of 200, not 4, and when $x = 0$, y is equal to 4, not 200.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID ecbfeafb

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: ecbfeafb

The function g is defined by $g(x) = x(x - 2)(x + 6)^2$. The value of $g(7 - w)$ is 0, where w is a constant. What is the sum of all possible values of w ?

ID: ecbfeafb Answer

Correct Answer: 25

Rationale

The correct answer is 25. The value of $g(7 - w)$ is the value of gx when $x = 7 - w$, where w is a constant. Substituting $7 - w$ for x in the given equation yields $g(7 - w) = (7 - w)(7 - w - 2)(7 - w + 6)^2$, which is equivalent to $g(7 - w) = (7 - w)(5 - w)(13 - w)^2$. It's given that the value of $g(7 - w)$ is 0. Substituting 0 for $g(7 - w)$ in the equation $g(7 - w) = (7 - w)(5 - w)(13 - w)^2$ yields $0 = (7 - w)(5 - w)(13 - w)^2$. Since the product of the three factors on the right-hand side of this equation is equal to 0, at least one of these three factors must be equal to 0. Therefore, the possible values of w can be found by setting each factor equal to 0. Setting the first factor equal to 0 yields $7 - w = 0$. Adding w to both sides of this equation yields $7 = w$. Therefore, 7 is one possible value of w . Setting the second factor equal to 0 yields $5 - w = 0$. Adding w to both sides of this equation yields $5 = w$. Therefore, 5 is a second possible value of w . Setting the third factor equal to 0 yields $13 - w^2 = 0$. Taking the square root of both sides of this equation yields $13 - w = 0$. Adding w to both sides of this equation yields $13 = w$. Therefore, 13 is a third possible value of w . Adding the three possible values of w yields $7 + 5 + 13$, or 25. Therefore, the sum of all possible values of w is 25.

Question Difficulty: Hard

Question ID 2009ef32

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 2009ef32

The product of two positive integers is **546**. If the first integer is **11** greater than twice the second integer, what is the smaller of the two integers?

- A. **7**
- B. **14**
- C. **39**
- D. **78**

ID: 2009ef32 Answer

Correct Answer: B

Rationale

Choice B is correct. Let x be the first integer and let y be the second integer. If the first integer is 11 greater than twice the second integer, then $x = 2y + 11$. If the product of the two integers is 546, then $xy = 546$. Substituting $2y + 11$ for x in this equation results in $2y + 11y = 546$. Distributing the y to both terms in the parentheses results in $2y^2 + 11y = 546$. Subtracting 546 from both sides of this equation results in $2y^2 + 11y - 546 = 0$. The left-hand side of this equation can be factored by finding two values whose product is 2-546, or -1,092, and whose sum is 11. The two values whose product is -1,092 and whose sum is 11 are 39 and -28. Thus, the equation $2y^2 + 11y - 546 = 0$ can be rewritten as $2y^2 + 28y - 39y - 546 = 0$, which is equivalent to $2yy - 14 + 39y - 14 = 0$, or $2y + 39y - 14 = 0$. By the zero product property, it follows that $2y + 39 = 0$ and $y - 14 = 0$. Subtracting 39 from both sides of the equation $2y + 39 = 0$ yields $2y = -39$. Dividing both sides of this equation by 2 yields $y = -\frac{39}{2}$. Since y is a positive integer, the value of y is not $-\frac{39}{2}$. Adding 14 to both sides of the equation $y - 14 = 0$ yields $y = 14$. Substituting 14 for y in the equation $xy = 546$ yields $x14 = 546$. Dividing both sides of this equation by 14 results in $x = 39$. Therefore, the two integers are 14 and 39, so the smaller of the two integers is 14.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the larger of the two integers.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2c94a7ab

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 2c94a7ab

$f(x) = (x + 6)(x + 5)(x - 4)$

The function f is given. Which table of values represents $y = f(x) - 3$?

A.

x	y
-6	-9
-5	-8
4	1

B.

x	y
-6	-3
-5	-3
4	-3

C.

x	y
-6	-3
-5	-2
4	7

D.

x	y
-6	3
-5	3
4	3

ID: 2c94a7ab Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that $fx = (x + 6)(x + 5)(x - 4)$ and $y = f(x) - 3$. Substituting $(x + 6)(x + 5)(x - 4)$ for fx in the equation $y = f(x) - 3$ yields $y = (x + 6)(x + 5)(x - 4) - 3$. Substituting -6 for x in this equation yields $y = (-6 + 6)(-6 + 5)(-6 - 4) - 3$, or $y = -3$. Substituting -5 for x in the equation $y = (x + 6)(x + 5)(x - 4) - 3$ yields $y = (-5 + 6)(-5 + 5)(-5 - 4) - 3$, or $y = -3$. Substituting 4 for x in the equation $y = (x + 6)(x + 5)(x - 4) - 3$ yields

$y = (4 + 6)(4 + 5)(4 - 4) - 3$, or $y = -3$. Therefore, when $x = -6$ then $y = -3$, when $x = -5$ then $y = -3$, and when $x = 4$ then $y = -3$. Thus, the table of values in choice B represents $y = f(x) - 3$.

Choice A is incorrect. This table represents $y = x - 3$ rather than $y = f(x) - 3$.

Choice C is incorrect. This table represents $y = x + 3$ rather than $y = f(x) - 3$.

Choice D is incorrect. This table represents $y = fx + 3$ rather than $y = f(x) - 3$.

Question Difficulty: Hard

Question ID 7c505685

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 7c505685

The function f is defined by $f(x) = a^x + b$, where a and b are constants. In the xy -plane, the graph of $y = f(x)$ has an x -intercept at $(2, 0)$ and a y -intercept at $(0, -323)$. What is the value of b ?

ID: 7c505685 Answer

Correct Answer: -324

Rationale

The correct answer is -324. It's given that the function f is defined by $fx = a^x + b$, where a and b are constants. It's also given that the graph of $y = fx$ has a y -intercept at 0, - 323. It follows that $f0 = - 323$. Substituting 0 for x and -323 for fx in $fx = a^x + b$ yields $-323 = a^0 + b$, or $-323 = 1 + b$. Subtracting 1 from each side of this equation yields $-324 = b$. Therefore, the value of b is -324.

Question Difficulty: Hard

Question ID a4f13e81

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: a4f13e81

$f(x) = 4x^2 + 64x + 262$

The function g is defined by $g(x) = f(x + 5)$. For what value of x does $g(x)$ reach its minimum?

- A. -13
- B. -8
- C. -5
- D. -3

ID: a4f13e81 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $gx = fx + 5$. Since $fx = 4x^2 + 64x + 262$, it follows that $fx + 5 = 4x + 5^2 + 64x + 5 + 262$. Expanding the quantity $x + 5^2$ in this equation yields $fx + 5 = 4x^2 + 10x + 25 + 64x + 5 + 262$. Distributing the 4 and the 64 yields $fx + 5 = 4x^2 + 40x + 100 + 64x + 320 + 262$. Combining like terms yields $fx + 5 = 4x^2 + 104x + 682$. Therefore, $gx = 4x^2 + 104x + 682$. For a quadratic function defined by an equation of the form $gx = ax - h^2 + k$, where a , h , and k are constants and a is positive, gx reaches its minimum, k , when the value of x is h . The equation $gx = 4x^2 + 104x + 682$ can be rewritten in this form by completing the square. This equation is equivalent to $gx = 4x^2 + 26 + 682$, or $gx = 4x^2 + 26x + 169 - 169 + 682$. This equation can be rewritten as $gx = 4x + 13^2 - 169 + 682$, or $gx = 4x + 13^2 - 4169 + 682$, which is equivalent to $gx = 4x + 13^2 + 6$. This equation is in the form $gx = ax - h^2 + k$, where $a = 4$, $h = -13$, and $k = 6$. Therefore, gx reaches its minimum when the value of x is -13 .

Choice B is incorrect. This is the value of x for which fx , rather than gx , reaches its minimum.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the value of x for which $fx - 5$, rather than $fx + 5$, reaches its minimum.

Question Difficulty: Hard

Question ID 065cc8fc

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 065cc8fc

The equation $E(t) = 5(1.8)^t$ gives the estimated number of employees at a restaurant, where t is the number of years since the restaurant opened. Which of the following is the best interpretation of the number 5 in this context?

- A. The estimated number of employees when the restaurant opened
- B. The increase in the estimated number of employees each year
- C. The number of years the restaurant has been open
- D. The percent increase in the estimated number of employees each year

ID: 065cc8fc Answer

Correct Answer: A

Rationale

Choice A is correct. For an exponential function of the form $Et = ab^t$, where a and b are constants, the initial value of the function—that is, the value of the function when $t = 0$ —is a and the value of the function increases by a factor of b each time t increases by 1. Since the function $Et = 51.8^t$ gives the estimated number of employees at a restaurant and t is the number of years since the restaurant opened, the best interpretation of the number 5 in this context is the estimated number of employees when $t = 0$, or when the restaurant opened.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Choice D is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 914f0fb6

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 914f0fb6

$$f(x) = 5,470(0.64)^{\frac{x}{12}}$$

The function f gives the value, in dollars, of a certain piece of equipment after x months of use. If the value of the equipment decreases each year by $p\%$ of its value the preceding year, what is the value of p ?

- A. 4
- B. 5
- C. 36
- D. 64

ID: 914f0fb6 Answer

Correct Answer: C

Rationale

Choice C is correct. For a function of the form $fx = ar^{\frac{x}{k}}$, where a , r , and k are constants and $r < 1$, the value of fx decreases by $100(1 - r)\%$ for every increase of x by k . In the given function, $a = 5,470$, $r = 0.64$, and $k = 12$. Therefore, for the given function, the value of fx decreases by $100(1 - 0.64)\%$, or 36% , for every increase of x by 12. Since fx represents the value, in dollars, of the equipment after x months of use, it follows that the value of the equipment decreases every 12 months by 36% of its value the preceding 12 months. Since there are 12 months in a year, the value of the equipment decreases each year by 36% of its value the preceding year. Thus, the value of p is 36.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 3c206d51

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 3c206d51

The function h is defined by $h(x) = a^x + b$, where a and b are positive constants. The graph of $y = h(x)$ in the xy -plane passes through the points $(0, 10)$ and $(-2, \frac{325}{36})$. What is the value of ab ?

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. 54
- D. 60

ID: 3c206d51 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function h is defined by $hx = a^x + b$ and that the graph of $y = h(x)$ in the xy -plane passes through the points $0, 10$ and $-2, \frac{325}{36}$. Substituting 0 for x and 10 for hx in the equation $hx = a^x + b$ yields $10 = a^0 + b$, or $10 = 1 + b$. Subtracting 1 from both sides of this equation yields $9 = b$. Substituting -2 for x and $\frac{325}{36}$ for hx in the equation $h(x) = a^x + 9$ yields $\frac{325}{36} = a^{-2} + 9$. Subtracting 9 from both sides of this equation yields $\frac{1}{36} = a^{-2}$, which can be rewritten as $a^2 = 36$. Taking the square root of both sides of this equation yields $a = 6$ and $a = -6$, but because it's given that a is a positive constant, a must equal 6 . Because the value of a is 6 and the value of b is 9 , the value of ab is $(6)(9)$, or 54 .

Choice A is incorrect and may result from finding the value of $a^{-2}b$ rather than the value of ab .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from correctly finding the value of a as 6 , but multiplying it by the y -value in the first ordered pair rather than by the value of b .

Question Difficulty: Hard

Question ID 1ca5e8ac

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 1ca5e8ac

The function f is defined by $f(x) = (-8)(2)^x + 22$. What is the y -intercept of the graph of $y = f(x)$ in the xy -plane?

- A. $(0, 14)$
- B. $(0, 2)$
- C. $(0, 22)$
- D. $(0, -8)$

ID: 1ca5e8ac Answer

Correct Answer: A

Rationale

Choice A is correct. The y -intercept of the graph of $y = fx$ in the xy -plane occurs at the point on the graph where $x = 0$. In other words, when $x = 0$, the corresponding value of fx is the y -coordinate of the y -intercept. Substituting 0 for x in the given equation yields $f0 = -82^0 + 22$, which is equivalent to $f0 = -81 + 22$, or $f0 = 14$. Thus, when $x = 0$, the corresponding value of fx is 14. Therefore, the y -intercept of the graph of $y = fx$ in the xy -plane is 0, 14.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This could be the y -intercept for $fx = -82^x$, not $fx = -82^x + 22$.

Question Difficulty: Hard

Question ID 7be44fda

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 7be44fda

x	$f(x)$
-1	10
0	14
1	20

For the quadratic function f , the table shows three values of x and their corresponding values of $f(x)$. Which equation defines f ?

- A. $f(x) = 3x^2 + 3x + 14$
- B. $f(x) = 5x^2 + x + 14$
- C. $f(x) = 9x^2 - x + 14$
- D. $f(x) = x^2 + 5x + 14$

ID: 7be44fda Answer

Correct Answer: D

Rationale

Choice D is correct. The equation of a quadratic function can be written in the form $fx = ax - h^2 + k$, where a , h , and k are constants. It's given in the table that when $x = -1$, the corresponding value of fx is 10. Substituting -1 for x and 10 for fx in the equation $fx = ax - h^2 + k$ gives $10 = a(-1) - h^2 + k$, which is equivalent to $10 = -a - h^2 + k$, or $10 = -a - 2h + h^2 + k$, or $10 = -a - 2h + h^2 + k$. It's given in the table that when $x = 0$, the corresponding value of fx is 14. Substituting 0 for x and 14 for fx in the equation $fx = ax - h^2 + k$ gives $14 = a(0) - h^2 + k$, or $14 = -h^2 + k$. It's given in the table that when $x = 1$, the corresponding value of fx is 20. Substituting 1 for x and 20 for fx in the equation $fx = ax - h^2 + k$ gives $20 = a(1) - h^2 + k$, which is equivalent to $20 = a - h^2 + k$, or $20 = a - 2h + h^2 + k$. Adding $20 = a - 2h + h^2 + k$ to the equation $10 = -a - 2h + h^2 + k$ gives $30 = 2a + 2ah^2 + 2k$. Dividing both sides of this equation by 2 gives $15 = a + ah^2 + k$. Since $14 = -h^2 + k$, substituting 14 for $ah^2 + k$ into the equation $15 = a + ah^2 + k$ gives $15 = a + 14$. Subtracting 14 from both sides of this equation gives $a = 1$. Substituting 1 for a in the equations $14 = -h^2 + k$ and $20 = a - 2h + h^2 + k$ gives $14 = -h^2 + k$ and $20 = 1 - 2h + h^2 + k$, respectively. Since $14 = -h^2 + k$, substituting 14 for $-h^2 + k$ in the equation $20 = 1 - 2h + h^2 + k$ gives $20 = 1 - 2h + 14$, or $20 = 15 - 2h$. Subtracting 15 from both sides of this equation gives $5 = -2h$. Dividing both sides of this equation by -2 gives $-\frac{5}{2} = h$. Substituting $-\frac{5}{2}$ for h into the equation $14 = -h^2 + k$ gives $14 = -\frac{25}{4} + k$, or $14 = \frac{25}{4} + k$. Subtracting $\frac{25}{4}$ from both sides of this equation gives $\frac{31}{4} = k$. Substituting 1 for a , $-\frac{5}{2}$ for h , and $\frac{31}{4}$ for k in the equation $fx = ax - h^2 + k$ gives $fx = x + \frac{5^2}{2} + \frac{31}{4}$, which is equivalent to $fx = x^2 + 5x + \frac{25}{4} + \frac{31}{4}$, or $fx = x^2 + 5x + 14$. Therefore, $fx = x^2 + 5x + 14$ defines f .

Choice A is incorrect. If $fx = 3x^2 + 3x + 14$, then when $x = -1$, the corresponding value of fx is 14, not 10.

Choice B is incorrect. If $fx = 5x^2 + x + 14$, then when $x = -1$, the corresponding value of fx is 18, not 10.

Choice C is incorrect. If $fx = 9x^2 - x + 14$, then when $x = -1$, the corresponding value of fx is 24, not 10, and when $x = 1$, the corresponding value of fx is 22, not 20.

Question Difficulty: Hard

Question ID 2f73623c

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 2f73623c

x	$g(x)$
-27	3
-9	0
21	5

The table shows three values of x and their corresponding values of $g(x)$, where $g(x) = \frac{f(x)}{x+3}$ and f is a linear function. What is the y -intercept of the graph of $y = f(x)$ in the xy -plane?

- A. (0, 36)
- B. (0, 12)
- C. (0, 4)
- D. (0, -9)

ID: 2f73623c Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the table shows values of x and their corresponding values of gx , where $gx = \frac{fx}{x+3}$. It's also given that f is a linear function. It follows that an equation that defines f can be written in the form $fx = mx + b$, where m represents the slope and b represents the y -coordinate of the y -intercept $0, b$ of the graph of $y = fx$ in the xy -plane. The slope of the graph of $y = fx$ can be found using two points, x_1, y_1 and x_2, y_2 , that are on the graph of $y = fx$, and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Since the table shows values of x and their corresponding values of gx , substituting values of x and gx in the equation $gx = \frac{fx}{x+3}$ can be used to define function f . Using the first pair of values from the table, $x = -27$ and $gx = 3$, yields $3 = \frac{f-27}{-27+3}$, or $3 = \frac{f-27}{-24}$. Multiplying each side of this equation by -24 yields $-72 = f-27$, so the point $-27, -72$ is on the graph of $y = fx$. Using the second pair of values from the table, $x = -9$ and $gx = 0$, yields $0 = \frac{f-9}{-9+3}$, or $0 = \frac{f-9}{-6}$. Multiplying each side of this equation by -6 yields $0 = f-9$, so the point $-9, 0$ is on the graph of $y = fx$. Substituting $-27, -72$ and $-9, 0$ for x_1, y_1 and x_2, y_2 , respectively, in the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ yields $m = \frac{0 - -72}{-9 - -27}$, or $m = 4$. Substituting 4 for m in the equation $fx = mx + b$ yields $fx = 4x + b$. Since $0 = f-9$, substituting -9 for x and 0 for fx in the equation $fx = 4x + b$ yields $0 = 4-9 + b$, or $0 = -36 + b$. Adding 36 to both sides of this equation yields $36 = b$. It follows that 36 is the y -coordinate of the y -intercept $0, b$ of the graph of $y = fx$. Therefore, the y -intercept of the graph of $y = fx$ is $0, 36$.

Choice B is incorrect. 12 is the y -coordinate of the y -intercept of the graph of $y = gx$.

Choice C is incorrect. 4 is the slope of the graph of $y = fx$.

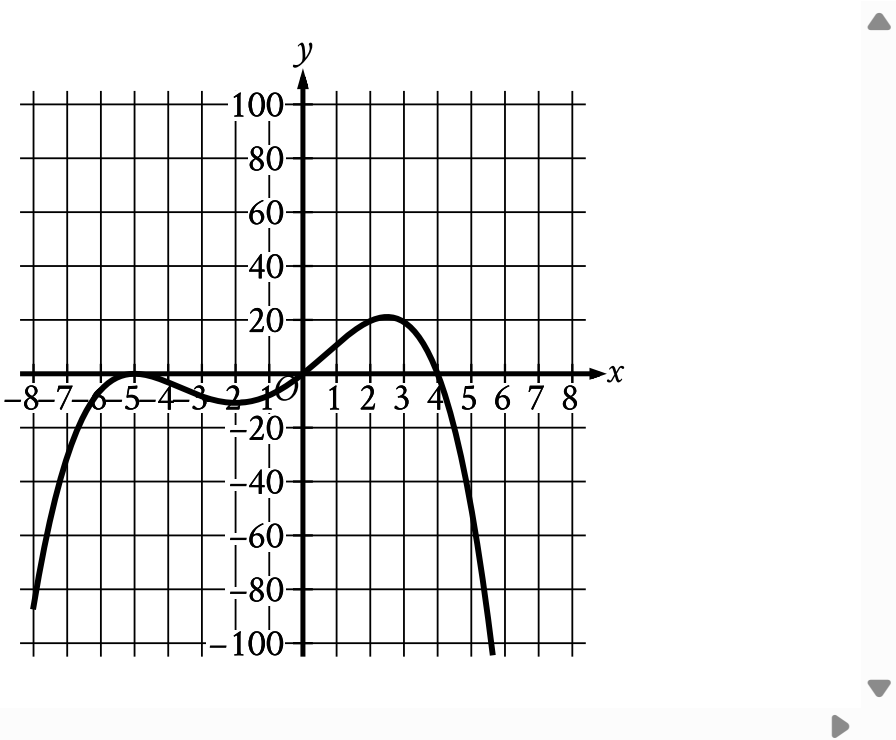
Choice D is incorrect. -9 is the x-coordinate of the x-intercept of the graph of $y = fx$.

Question Difficulty: Hard

Question ID ed951a6b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: ed951a6b



Which of the following could be the equation of the graph shown in the xy -plane?

- A. $y = -\frac{1}{10}x(x - 4)(x + 5)$
- B. $y = -\frac{1}{10}x(x - 4)(x + 5)^2$
- C. $y = -\frac{1}{10}x(x - 5)(x + 4)$
- D. $y = -\frac{1}{10}x(x - 5)^2(x + 4)$

ID: ed951a6b Answer

Correct Answer: B

Rationale

Choice B is correct. Each of the given choices is an equation of the form $y = -\frac{1}{10}xx - a^mx + b^n$, where a, b, m , and n are positive constants. In the xy -plane, the graph of an equation of this form has x -intercepts at $x = 0, x = a$, and $x = -b$. The graph shown has x -intercepts at $x = 0, x = 4$, and $x = -5$. Therefore, $a = 4$ and $b = 5$. Of the given choices, only choices A and B have $a = 4$ and $b = 5$. For an equation in the form $y = -\frac{1}{10}xx - a^mx + b^n$, if all values of x that are less than $-b$ or greater than a correspond to negative y -values, then the sum of all the exponents of the factors on the right-hand side of the equation is even. In the graph shown, all values of x less than -5 or greater than 4 correspond to negative y -values. Therefore, the sum of all the exponents of the factors on the right-hand side of the equation $y = -\frac{1}{10}xx - 4^mx + 5^n$ must be even. For choice A, the sum of these exponents is

$1 + 1 + 1$, or 3, which is odd. For choice B, the sum of these exponents is $1 + 1 + 2$, or 4, which is even. Therefore, $y = -\frac{1}{10}xx - 4x + 5^2$ could be the equation of the graph shown.

Choice A is incorrect. For the graph of this equation, all values of x less than -5 correspond to positive, not negative, y -values.

Choice C is incorrect. The graph of this equation has x -intercepts at $x = -4$, $x = 0$, and $x = 5$, rather than x -intercepts at $x = -5$, $x = 0$, and $x = 4$.

Choice D is incorrect. The graph of this equation has x -intercepts at $x = -4$, $x = 0$, and $x = 5$, rather than x -intercepts at $x = -5$, $x = 0$, and $x = 4$.

Question Difficulty: Hard

Question ID 685e4f6b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 685e4f6b

The function $f(t) = 60,000(2)^{\frac{t}{410}}$ gives the number of bacteria in a population t minutes after an initial observation. How much time, in minutes, does it take for the number of bacteria in the population to double?

ID: 685e4f6b Answer

Correct Answer: 410

Rationale

The correct answer is 410. It's given that t minutes after an initial observation, the number of bacteria in a population is $60,0002^{\frac{t}{410}}$. This expression consists of the initial number of bacteria, 60,000, multiplied by the expression $2^{\frac{t}{410}}$. The time it takes for the number of bacteria to double is the increase in the value of t that causes the expression $2^{\frac{t}{410}}$ to double. Since the base of the expression $2^{\frac{t}{410}}$ is 2, the expression $2^{\frac{t}{410}}$ will double when the exponent increases by 1. Since the exponent of the expression $2^{\frac{t}{410}}$ is $\frac{t}{410}$, the exponent will increase by 1 when t increases by 410. Therefore the time, in minutes, it takes for the number of bacteria in the population to double is 410.

Question Difficulty: Hard

Question ID 877585e8

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 877585e8

A model estimates that at the end of each year from **2015** to **2020**, the number of squirrels in a population was **150%** more than the number of squirrels in the population at the end of the previous year. The model estimates that at the end of **2016**, there were **180** squirrels in the population. Which of the following equations represents this model, where n is the estimated number of squirrels in the population t years after the end of **2015** and $t \leq 5$?

- A. $n = 72(1.5)^t$
- B. $n = 72(2.5)^t$
- C. $n = 180(1.5)^t$
- D. $n = 180(2.5)^t$

ID: 877585e8 Answer

Correct Answer: B

Rationale

Choice B is correct. Since the model estimates that the number of squirrels in the population increased by a fixed percentage, 150%, each year, the model can be represented by an exponential equation of the form $n = a1 + \frac{p}{100}^t$, where a is the estimated number of squirrels in the population at the end of 2015, and the model estimates that at the end of each year, the number is $p\%$ more than the number at the end of the previous year. Since the model estimates that at the end of each year, the number was 150% more than the number at the end of the previous year, $p = 150$. Substituting 150 for p in the equation $n = a1 + \frac{p}{100}^t$ yields $n = a1 + \frac{150}{100}^t$, which is equivalent to $n = a1 + 1.5^t$, or $n = a2.5^t$. It's given that the estimated number of squirrels at the end of 2016 was 180. This means that when $t = 1$, $n = 180$. Substituting 1 for t and 180 for n in the equation $n = a2.5^t$ yields $180 = a2.5^1$, or $180 = 2.5a$. Dividing each side of this equation by 2.5 yields $72 = a$. Substituting 72 for a in the equation $n = a2.5^t$ yields $n = 722.5^t$.

Choice A is incorrect. This equation represents a model where at the end of each year, the estimated number of squirrels was 150% of, not 150% more than, the estimated number at the end of the previous year.

Choice C is incorrect. This equation represents a model where at the end of each year, the estimated number of squirrels was 150% of, not 150% more than, the estimated number at the end of the previous year, and the estimated number of squirrels at the end of 2015, not the end of 2016, was 180.

Choice D is incorrect. This equation represents a model where the estimated number of squirrels at the end of 2015, not the end of 2016, was 180.

Question Difficulty: Hard

Question ID 42bc647d

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 42bc647d

A machine launches a softball from ground level. The softball reaches a maximum height of **51.84** meters above the ground at **1.8** seconds and hits the ground at **3.6** seconds. Which equation represents the height above ground h , in meters, of the softball t seconds after it is launched?

- A. $h = -t^2 + 3.6$
- B. $h = -t^2 + 51.84$
- C. $h = -16(t - 1.8)^2 - 3.6$
- D. $h = -16(t - 1.8)^2 + 51.84$

ID: 42bc647d Answer

Correct Answer: D

Rationale

Choice D is correct. An equation representing the height above ground h , in meters, of a softball t seconds after it is launched by a machine from ground level can be written in the form $h = -at - b^2 + c$, where a , b , and c are positive constants. In this equation, b represents the time, in seconds, at which the softball reaches its maximum height of c meters above the ground. It's given that this softball reaches a maximum height of 51.84 meters above the ground at 1.8 seconds; therefore, $b = 1.8$ and $c = 51.84$. Substituting 1.8 for b and 51.84 for c in the equation $h = -at - b^2 + c$ yields $h = -at - 1.8^2 + 51.84$. It's also given that this softball hits the ground at 3.6 seconds; therefore, $h = 0$ when $t = 3.6$. Substituting 0 for h and 3.6 for t in the equation $h = -at - 1.8^2 + 51.84$ yields $0 = -a3.6 - 1.8^2 + 51.84$, which is equivalent to $0 = -a1.8^2 + 51.84$, or $0 = -3.24a + 51.84$. Adding $3.24a$ to both sides of this equation yields $3.24a = 51.84$. Dividing both sides of this equation by 3.24 yields $a = 16$. Substituting 16 for a in the equation $h = -at - 1.8^2 + 51.84$ yields $h = -16t - 1.8^2 + 51.84$. Therefore, $h = -16t - 1.8^2 + 51.84$ represents the height above ground h , in meters, of this softball t seconds after it is launched.

Choice A is incorrect. This equation represents a situation where the maximum height is 3.6 meters above the ground at 0 seconds, not 51.84 meters above the ground at 1.8 seconds.

Choice B is incorrect. This equation represents a situation where the maximum height is 51.84 meters above the ground at 0 seconds, not 1.8 seconds.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 6d44de27

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 6d44de27

$f(x) = ax^2 + 4x + c$

In the given quadratic function, a and c are constants. The graph of $y = f(x)$ in the xy -plane is a parabola that opens upward and has a vertex at the point (h, k) , where h and k are constants. If $k < 0$ and $f(-9) = f(3)$, which of the following must be true?

- I. $c < 0$

II. $a \geq 1$
- A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 6d44de27 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of $y = fx$ in the xy -plane is a parabola with vertex h, k . If $f-9 = f3$, then for the graph of $y = fx$, the point with an x -coordinate of -9 and the point with an x -coordinate of 3 have the same y -coordinate. In the xy -plane, a parabola is a symmetric graph such that when two points have the same y -coordinate, these points are equidistant from the vertex, and the x -coordinate of the vertex is halfway between the x -coordinates of these two points. Therefore, for the graph of $y = fx$, the points with x -coordinates -9 and 3 are equidistant from the vertex, h, k , and h is halfway between -9 and 3 . The value that is halfway between -9 and 3 is $\frac{-9+3}{2}$, or -3 . Therefore, $h = -3$. The equation defining f can also be written in vertex form, $fx = ax - h^2 + k$. Substituting -3 for h in this equation yields $fx = ax - (-3)^2 + k$, or $fx = ax + 3^2 + k$. This equation is equivalent to $fx = ax^2 + 6x + 9 + k$, or $fx = ax^2 + 6ax + 9a + k$. Since $fx = ax^2 + 4x + c$, it follows that $6a = 4$ and $9a + k = c$. Dividing both sides of the equation $6a = 4$ by 6 yields $a = \frac{4}{6}$, or $a = \frac{2}{3}$. Since $\frac{2}{3} < 1$, it's not true that $a \geq 1$. Therefore, statement II isn't true. Substituting $\frac{2}{3}$ for a in the equation $9a + k = c$ yields $9\frac{2}{3} + k = c$, or $6 + k = c$. Subtracting 6 from both sides of this equation yields $k = c - 6$. If $k < 0$, then $c - 6 < 0$, or $c < 6$. Since c could be any value less than 6 , it's not necessarily true that $c < 0$. Therefore, statement I isn't necessarily true. Thus, neither I nor II must be true.

- Choice A is incorrect and may result from conceptual or calculation errors.
- Choice B is incorrect and may result from conceptual or calculation errors.
- Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 31041ee4

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 31041ee4

The function g is defined by $g(x) = (x + 14)(t - x)$, where t is a constant. In the xy -plane, the graph of $y = g(x)$ passes through the point $(24, 0)$. What is the value of $g(0)$?

ID: 31041ee4 Answer

Correct Answer: 336

Rationale

The correct answer is 336. By the zero product property, if $x + 14t - x = 0$, then $x + 14 = 0$, which gives $x = -14$, or $t - x = 0$, which gives $x = t$. Therefore, $gx = 0$ when $x = -14$ and when $x = t$. Since the graph of $y = gx$ passes through the point $24, 0$, it follows that $g24 = 0$, so $t = 24$. Substituting 24 for t in the equation $gx = x + 14t - x$ yields $gx = x + 1424 - x$. The value of $g0$ can be calculated by substituting 0 for x in this equation, which yields $g0 = 0 + 1424 - 0$, or $g0 = 336$.

Question Difficulty: Hard

Question ID 7dea2114

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 7dea2114

Immanuel purchased a certain rare coin on January 1. The function $f(x) = 65(1.03)^x$, where $0 \leq x \leq 10$, gives the predicted value, in dollars, of the rare coin x years after Immanuel purchased it. What is the best interpretation of the statement “ $f(8)$ is approximately equal to 82” in this context?

- A. When the rare coin's predicted value is approximately 82 dollars, it is 8% greater than the predicted value, in dollars, on January 1 of the previous year.
- B. When the rare coin’s predicted value is approximately 82 dollars, it is 8 times the predicted value, in dollars, on January 1 of the previous year.
- C. From the day Immanuel purchased the rare coin to 8 years after Immanuel purchased the coin, its predicted value increased by a total of approximately 82 dollars.
- D. 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

ID: 7dea2114 Answer

Correct Answer: D

Rationale

Choice D is correct. It’s given that the function $f(x) = 65(1.03)^x$ gives the predicted value, in dollars, of a certain rare coin x years after Immanuel purchased it. It follows that $f(x)$ represents the predicted value, in dollars, of the coin x years after Immanuel purchased it. Since the value of $f(8)$ is the value of $f(x)$ when $x = 8$, it follows that “ $f(8)$ is approximately equal to 82” means that $f(x)$ is approximately equal to 82 when $x = 8$. Therefore, the best interpretation of the statement “ $f(8)$ is approximately equal to 82” in this context is 8 years after Immanuel purchased the rare coin, its predicted value is approximately 82 dollars.

Choice A is incorrect and may result from conceptual errors.

Choice B is incorrect and may result from conceptual errors.

Choice C is incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID 877bd007

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 877bd007

$f(x) = 4x^2 - 50x + 126$

The given equation defines the function f . For what value of x does $f(x)$ reach its minimum?

ID: 877bd007 Answer

Correct Answer: 25/4, 6.25

Rationale

The correct answer is $\frac{25}{4}$. The given equation can be rewritten in the form $fx = ax - h^2 + k$, where a , h , and k are constants. When $a > 0$, h is the value of x for which fx reaches its minimum. The given equation can be rewritten as $fx = 4x^2 - \frac{50}{4}x + 126$, which is equivalent to $fx = 4x^2 - \frac{50}{4}x + \frac{50^2}{8} - \frac{50^2}{8} + 126$. This equation can be rewritten as $fx = 4x - \frac{50^2}{8} - \frac{50^2}{8} + 126$, or $fx = 4x - \frac{50^2}{8} - 4\frac{50^2}{8} + 126$, which is equivalent to $fx = 4x - \frac{25^2}{4} - \frac{121}{4}$. Therefore, $h = \frac{25}{4}$, so the value of x for which fx reaches its minimum is $\frac{25}{4}$. Note that 25/4 and 6.25 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 2584d355

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 2584d355

Which of the following functions has(have) a minimum value at -3 ?

I. $f(x) = -6(3)^x - 3$

II. $g(x) = -3(6)^x$

- A. I only
- B. II only
- C. I and II
- D. Neither I nor II

ID: 2584d355 Answer

Correct Answer: D

Rationale

Choice D is correct. A function of the form $fx = ab^x + c$, where $a < 0$ and $b > 1$, is a decreasing function. Both of the given functions are of this form; therefore, both are decreasing functions. If a function f is decreasing as the value of x increases, the corresponding value of fx decreases; therefore, the function doesn't have a minimum value. Thus, neither of the given functions has a minimum value.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 62ba1727

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 62ba1727

$$y = x^2 - 14x + 22$$

The given equation relates the variables x and y . For what value of x does the value of y reach its minimum?

ID: 62ba1727 Answer

Correct Answer: 7

Rationale

The correct answer is 7. When an equation is of the form $y = ax^2 + bx + c$, where a , b , and c are constants, the value of y reaches its minimum when $x = -\frac{b}{2a}$. Since the given equation is of the form $y = ax^2 + bx + c$, it follows that $a = 1$, $b = -14$, and $c = 22$. Therefore, the value of y reaches its minimum when $x = -\frac{-14}{2(1)}$, or $x = 7$.

Question Difficulty: Hard

Question ID 62821aef

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 62821aef

$f(x) = (x - 14)(x + 19)$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

- A. -266
- B. -19
- C. $-\frac{33}{2}$
- D. $-\frac{5}{2}$

ID: 62821aef Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $fx = x - 14x + 19$, which can be rewritten as $fx = x^2 + 5x - 266$. Since the coefficient of the x^2 -term is positive, the graph of $y = fx$ in the xy -plane opens upward and reaches its minimum value at its vertex. The x -coordinate of the vertex is the value of x such that fx reaches its minimum. For an equation in the form $fx = ax^2 + bx + c$, where a , b , and c are constants, the x -coordinate of the vertex is $-\frac{b}{2a}$. For the equation $fx = x^2 + 5x - 266$, $a = 1$, $b = 5$, and $c = -266$. It follows that the x -coordinate of the vertex is $-\frac{5}{2(1)}$, or $-\frac{5}{2}$. Therefore, fx reaches its minimum when the value of x is $-\frac{5}{2}$.

Alternate approach: The value of x for the vertex of a parabola is the x -value of the midpoint between the two x -intercepts of the parabola. Since it's given that $fx = x - 14x + 19$, it follows that the two x -intercepts of the graph of $y = fx$ in the xy -plane occur when $x = 14$ and $x = -19$, or at the points $14, 0$ and $-19, 0$. The midpoint between two points, x_1, y_1 and x_2, y_2 , is $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$. Therefore, the midpoint between $14, 0$ and $-19, 0$ is $\frac{14 + (-19)}{2}, \frac{0 + 0}{2}$, or $-\frac{5}{2}, 0$. It follows that fx reaches its minimum when the value of x is $-\frac{5}{2}$.

Choice A is incorrect. This is the y -coordinate of the y -intercept of the graph of $y = fx$ in the xy -plane.

Choice B is incorrect. This is one of the x -coordinates of the x -intercepts of the graph of $y = fx$ in the xy -plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 5628913b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 5628913b

At the time that an article was first featured on the home page of a news website, there were **40** comments on the article. An exponential model estimates that at the end of each hour after the article was first featured on the home page, the number of comments on the article had increased by **190%** of the number of comments on the article at the end of the previous hour. Which of the following equations best represents this model, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and $t \leq 4$?

- A. $C = 40(1.19)^t$
- B. $C = 40(1.9)^t$
- C. $C = 40(19)^t$
- D. $C = 40(2.9)^t$

ID: 5628913b Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that an exponential model estimates that the number of comments on an article increased by a fixed percentage at the end of each hour. Therefore, the model can be represented by an exponential equation of the form $C = Ka^t$, where C is the estimated number of comments on the article t hours after the article was first featured on the home page and K and a are constants. It's also given that when the article was first featured on the home page of the news website, there were 40 comments on the article. This means that when $t = 0$, $C = 40$. Substituting 0 for t and 40 for C in the equation $C = Ka^t$ yields $40 = Ka^0$, or $40 = K$. It's also given that the number of comments on the article at the end of an hour had increased by 190% of the number of comments on the article at the end of the previous hour. Multiplying the percent increase by the number of comments on the article at the end of the previous hour yields the number of estimated additional comments the article has on its home page: $40 \frac{190}{100}$, or 76 comments. Thus, the estimated number of comments for the following hour is the sum of the comments from the end of the previous hour and the number of additional comments, which is $40 + 76$, or 116. This means that when $t = 1$, $C = 116$. Substituting 1 for t , 116 for C , and 40 for K in the equation $C = Ka^t$ yields $116 = 40a^1$, or $116 = 40a$. Dividing both sides of this equation by 40 yields $2.9 = a$. Substituting 40 for K and 2.9 for a in the equation $C = Ka^t$ yields $C = 40(2.9)^t$. Thus, the equation that best represents this model is $C = 40(2.9)^t$.

Choice A is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 19% of the number of comments at the end of the previous hour, rather than 190%.

Choice B is incorrect. This model represents a situation where the number of comments at the end of each hour increased by 90% of the number of comments at the end of the previous hour, rather than 190%.

Choice C is incorrect. This model represents a situation where the number of comments at the end of each hour was 19 times the number of comments at the end of the previous hour, rather than increasing by 190% of the number of comments at the end of the previous hour.

Question ID 40d75cea

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 40d75cea

$$f(t) = 8,000(0.65)^t$$

The given function f models the number of coupons a company sent to their customers at the end of each year, where t represents the number of years since the end of **1998**, and $0 \leq t \leq 5$. If $y = f(t)$ is graphed in the ty -plane, which of the following is the best interpretation of the y -intercept of the graph in this context?

- A. The minimum estimated number of coupons the company sent to their customers during the **5** years was **1,428**.
- B. The minimum estimated number of coupons the company sent to their customers during the **5** years was **8,000**.
- C. The estimated number of coupons the company sent to their customers at the end of **1998** was **1,428**.
- D. The estimated number of coupons the company sent to their customers at the end of **1998** was **8,000**.

ID: 40d75cea Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of a graph in the ty -plane is the point where $t = 0$. For the given function f , the y -intercept of the graph of $y = ft$ in the ty -plane can be found by substituting 0 for t in the equation $y = 8,0000 \cdot 65^t$, which gives $y = 8,0000 \cdot 65^0$. This is equivalent to $y = 8,0001$, or $y = 8,000$. Therefore, the y -intercept of the graph of $y = ft$ is 0,8,000. It's given that the function f models the number of coupons a company sent to their customers at the end of each year. Therefore, ft represents the estimated number of coupons the company sent to their customers at the end of each year. It's also given that t represents the number of years since the end of 1998. Therefore, $t = 0$ represents 0 years since the end of 1998, or the end of 1998. Thus, the best interpretation of the y -intercept of the graph of $y = ft$ is that the estimated number of coupons the company sent to their customers at the end of 1998 was 8,000.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID eae04f9

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: eae04f9

The function f is defined by $f(x) = a\sqrt{x + b}$, where a and b are constants. In the xy -plane, the graph of $y = f(x)$ passes through the point $(-24, 0)$, and $f(24) < 0$. Which of the following must be true?

- A. $f(0) = 24$
- B. $f(0) = -24$
- C. $a > b$
- D. $a < b$

ID: eae04f9 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(24) < 0$. Substituting 24 for x in the equation $y = a\sqrt{x + b}$ yields $f(24) = a\sqrt{24 + b}$. Therefore, $a\sqrt{24 + b} < 0$. Since $\sqrt{24 + b}$ can't be negative, it follows that $a < 0$. It's also given that the graph of $y = f(x)$ passes through the point $(-24, 0)$. It follows that when $x = -24$, $f(x) = 0$. Substituting -24 for x and 0 for $f(x)$ in the equation $y = a\sqrt{x + b}$ yields $0 = a\sqrt{-24 + b}$. By the zero product property, either $a = 0$ or $\sqrt{-24 + b} = 0$. Since $a < 0$, it follows that $\sqrt{-24 + b} = 0$. Squaring both sides of this equation yields $-24 + b = 0$. Adding 24 to both sides of this equation yields $b = 24$. Since $a < 0$ and b is 24, it follows that $a < b$ must be true.

Choice A is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which must be negative.

Choice B is incorrect. The value of $f(0)$ is $a\sqrt{b}$, which could be -24 , but doesn't have to be.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b589168d

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: b589168d

The exponential function g is defined by $g(x) = 19 \cdot a^x$, where a is a positive constant. If $g(3) = 2,375$, what is the value of $g(4)$?

ID: b589168d Answer

Correct Answer: 11875

Rationale

The correct answer is 11,875. It's given that the exponential function g is defined by $gx = 19 \cdot a^x$, where a is a positive constant, and $g3 = 2,375$. It follows that when $x = 3$, $gx = 2,375$. Substituting 3 for x and 2,375 for gx in the given equation yields $2,375 = 19 \cdot a^3$. Dividing each side of this equation by 19 yields $125 = a^3$. Taking the cube root of both sides of this equation gives $a = 5$. Substituting 4 for x and 5 for a in the equation $gx = 19 \cdot a^x$ yields $g4 = 19 \cdot 5^4$, or $g4 = 11,875$. Therefore, the value of $g4$ is 11,875.

Question Difficulty: Hard

Question ID 70cdc6d2

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 70cdc6d2

A sample of a certain isotope takes **29** years to decay to half its original mass. The function $s(t) = 184(0.5)^{\frac{t}{29}}$ gives the approximate mass of this isotope, in grams, that remains t years after a **184**-gram sample starts to decay. Which statement is the best interpretation of $s(87) = 23$ in this context?

- A. Approximately **23** grams of the sample remains **87** years after the sample starts to decay.
- B. The mass of the sample has decreased by approximately **23** grams **87** years after the sample starts to decay.
- C. The mass of the sample has decreased by approximately **87** grams **23** years after the sample starts to decay.
- D. Approximately **87** grams of the sample remains **23** years after the sample starts to decay.

ID: 70cdc6d2 Answer

Correct Answer: A

Rationale

Choice A is correct. In the given function, st represents the approximate mass, in grams, of the sample that remains t years after the sample starts to decay. It follows that the best interpretation of $s87 = 23$ is that approximately 23 grams of the sample remains 87 years after the sample starts to decay.

Choice B is incorrect. The mass of the sample has decreased by approximately $184 - 23$, or 161, grams, not 23 grams, 87 years after the sample starts to decay.

Choice C is incorrect. The mass of the sample has decreased by approximately 78 grams, not 87 grams, 23 years after the sample starts to decay.

Choice D is incorrect. This would be the best interpretation of $s23 = 87$, not $s87 = 23$.

Question Difficulty: Hard

Question ID 077b2830

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 077b2830

A scientist initially measures **12,000** bacteria in a growth medium. **4** hours later, the scientist measures **24,000** bacteria. Assuming exponential growth, the formula $P = C(2)^{rt}$ gives the number of bacteria in the growth medium, where r and C are constants and P is the number of bacteria t hours after the initial measurement. What is the value of r ?

- A. $\frac{1}{12,000}$
- B. $\frac{1}{4}$
- C. **4**
- D. **12,000**

ID: 077b2830 Answer

Correct Answer: B

Rationale

Choice B is correct. It’s given that the formula $P = C2^{rt}$ gives the number of bacteria in a growth medium, where r and C are constants and P is the number of bacteria t hours after the initial measurement. It’s also given that a scientist initially measures 12,000 bacteria in the growth medium. Since the initial measurement is 0 hours after the initial measurement, it follows that when $t = 0, P = 12,000$. Substituting 0 for t and 12,000 for P in the given equation yields $12,000 = C2^{r0}$, or $12,000 = C2^0$, which is equivalent to $12,000 = C$. It’s given that 4 hours later, the scientist measures 24,000 bacteria, or when $t = 4, P = 24,000$. Substituting 4 for t , 24,000 for P , and 12,000 for C in the given equation yields $24,000 = 12,0002^{4r}$. Dividing each side of this equation by 12,000 yields $2 = 2^{4r}$, or $2^1 = 2^{4r}$, which is equivalent to $1 = 4r$. Dividing both sides of this equation by 4 yields $\frac{1}{4} = r$. Therefore, the value of r is $\frac{1}{4}$.

Choice A is incorrect. This is the value of the reciprocal of C .

Choice C is incorrect. This is the value of the reciprocal of r .

Choice D is incorrect. This is the value of C .

Question Difficulty: Hard

Question ID 99a754e3

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 99a754e3

$$f(x) = (x - 1)(x + 3)(x - 2)$$

In the xy -plane, when the graph of the function f , where $y = f(x)$, is shifted up **6** units, the resulting graph is defined by the function g . If the graph of $y = g(x)$ crosses through the point $(4, b)$, where b is a constant, what is the value of b ?

ID: 99a754e3 Answer

Correct Answer: 48

Rationale

The correct answer is 48. It's given that in the xy -plane, when the graph of the function f , where $y = f(x)$, is shifted up 6 units, the resulting graph is defined by the function g . Therefore, function g can be defined by the equation $g(x) = f(x) + 6$. It's given that $f(x) = x^3 - 1x^2 + 3x - 2$. Substituting $x^3 - 1x^2 + 3x - 2$ for $f(x)$ in the equation $g(x) = f(x) + 6$ yields $g(x) = x^3 - 1x^2 + 3x - 2 + 6$. For the point $(4, b)$, the value of x is 4. Substituting 4 for x in the equation $g(x) = x^3 - 1x^2 + 3x - 2 + 6$ yields $g(4) = 4^3 - 1(4)^2 + 3(4) - 2 + 6$, or $g(4) = 48$. It follows that the graph of $y = g(x)$ crosses through the point $(4, 48)$. Therefore, the value of b is 48.

Question Difficulty: Hard

Question ID cfee492e

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: cfee492e

$y = 576^{(2x+2)}$

The graph of the given equation in the xy -plane has a y -intercept of (r, s) . Which of the following equivalent equations displays the value of s as a constant, a coefficient, or the base?

- A. $y = 331,776^{(x+1)}$
- B. $y = 24^{(4x+4)}$
- C. $y = \frac{1}{24}(24)^{(4x+5)}$
- D. $y = \frac{1}{576}(576)^{(2x+3)}$

ID: cfee492e Answer

Correct Answer: A

Rationale

Choice A is correct. The y -intercept of a graph in the xy -plane is the point where $x = 0$. Substituting 0 for x in the given equation, $y = 576^{2x+2}$, yields $y = 576^{20+2}$, which is equivalent to $y = 576^2$, or $y = 331,776$. Therefore, the graph of the given equation in the xy -plane has a y -intercept of 0, 331,776. It follows that $r = 0$ and $s = 331,776$. Thus, the equivalent equation $y = 331,776^{x+1}$ displays the value of s as the base.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

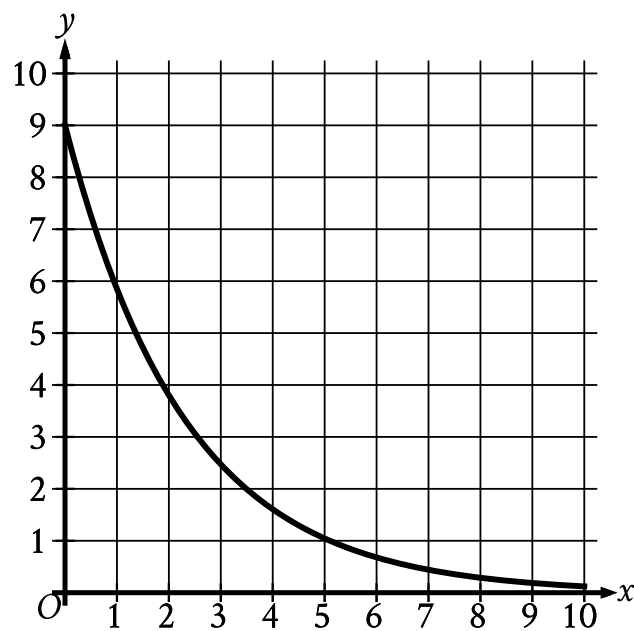
Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 92280623

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 92280623



The graph gives the estimated number of catalogs y , in thousands, a company sent to its customers at the end of each year, where x represents the number of years since the end of **1992**, where $0 \leq x \leq 10$. Which statement is the best interpretation of the y -intercept in this context?

- A. The estimated total number of catalogs the company sent to its customers during the first **10** years was **9,000**.
- B. The estimated total number of catalogs the company sent to its customers from the end of **1992** to the end of **2002** was **90**.
- C. The estimated number of catalogs the company sent to its customers at the end of **1992** was **9**.
- D. The estimated number of catalogs the company sent to its customers at the end of **1992** was **9,000**.

ID: 92280623 Answer

Correct Answer: D

Rationale

Choice D is correct. The y -intercept of the graph is the point at which the graph crosses the y -axis, or the point for which the value of x is 0. Therefore, the y -intercept of the given graph is the point 0, 9. It's given that x represents the number of years since the end of 1992. Therefore, $x = 0$ represents 0 years since the end of 1992, which is the same as the end of 1992. It's also given that y represents the estimated number of catalogs, in thousands, that the company sent to its customers at the end of the year. Therefore, $y = 9$ represents 9,000 catalogs. It follows that the y -intercept 0, 9 means that the estimated number of catalogs the company sent to its customers at the end of 1992 was 9,000.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a5dc10ab

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: a5dc10ab

$$p(x) + 57 = x^2$$

The given equation relates the value of x and its corresponding value of $p(x)$ for the function p . What is the minimum value of the function p ?

- A. $-3,249$
- B. -57
- C. 57
- D. $3,249$

ID: a5dc10ab Answer

Correct Answer: B

Rationale

Choice B is correct. For a quadratic function defined by an equation of the form $px = ax - h^2 + k$, where a , h , and k are constants and $a > 0$, the minimum value of the function is k . Subtracting 57 from both sides of the given equation yields $px = x^2 - 57$. This function is in the form $px = ax - h^2 + k$, where $a = 1$, $h = 0$, and $k = -57$. Therefore, the minimum value of the function p is -57.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 7d86a6ad

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 7d86a6ad

$f(x) = (x - 10)(x + 13)$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

- A. -130
- B. -13
- C. $-\frac{23}{2}$
- D. $-\frac{3}{2}$

ID: 7d86a6ad Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $fx = x - 10x + 13$, which can be rewritten as $fx = x^2 + 3x - 130$. Since the coefficient of the x^2 -term is positive, the graph of $y = fx$ in the xy -plane opens upward and reaches its minimum value at its vertex. The x -coordinate of the vertex is the value of x such that fx reaches its minimum. For an equation in the form $fx = ax^2 + bx + c$, where a , b , and c are constants, the x -coordinate of the vertex is $-\frac{b}{2a}$. For the equation $fx = x^2 + 3x - 130$, $a = 1$, $b = 3$, and $c = -130$. It follows that the x -coordinate of the vertex is $-\frac{3}{2(1)}$, or $-\frac{3}{2}$. Therefore, fx reaches its minimum when the value of x is $-\frac{3}{2}$.

Alternate approach: The value of x for the vertex of a parabola is the x -value of the midpoint between the two x -intercepts of the parabola. Since it's given that $fx = x - 10x + 13$, it follows that the two x -intercepts of the graph of $y = fx$ in the xy -plane occur when $x = 10$ and $x = -13$, or at the points $10, 0$ and $-13, 0$. The midpoint between two points, x_1, y_1 and x_2, y_2 , is $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$. Therefore, the midpoint between $10, 0$ and $-13, 0$ is $\frac{10 + (-13)}{2}, \frac{0 + 0}{2}$, or $-\frac{3}{2}, 0$. It follows that fx reaches its minimum when the value of x is $-\frac{3}{2}$.

Choice A is incorrect. This is the y -coordinate of the y -intercept of the graph of $y = fx$ in the xy -plane.

Choice B is incorrect. This is one of the x -coordinates of the x -intercepts of the graph of $y = fx$ in the xy -plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 1845987d

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 1845987d

$$f(x) = (x + 7)^2 + 4$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

ID: 1845987d Answer

Correct Answer: -7

Rationale

The correct answer is -7. For a quadratic function defined by an equation of the form $fx = ax - h^2 + k$, where a , h , and k are constants and $a > 0$, the function reaches its minimum when $x = h$. In the given function, $a = 1$, $h = -7$, and $k = 4$. Therefore, the value of x for which fx reaches its minimum is -7.

Question Difficulty: Hard

Question ID 77c8bd4a

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 77c8bd4a

For the function f , $f(0) = 86$, and for each increase in x by 1, the value of $f(x)$ decreases by 80%. What is the value of $f(2)$?

ID: 77c8bd4a Answer

Correct Answer: 3.44, 86/25

Rationale

The correct answer is 3.44. It's given that $f0 = 86$ and that for each increase in x by 1, the value of fx decreases by 80%. Because the output of the function decreases by a constant percentage for each 1-unit increase in the value of x , this relationship can be represented by an exponential function of the form $fx = ab^x$, where a represents the initial value of the function and b represents the rate of decay, expressed as a decimal. Because $f0 = 86$, the value of a must be 86. Because the value of fx decreases by 80% for each 1-unit increase in x , the value of b must be $(1 - 0.80)$, or 0.2. Therefore, the function f can be defined by $fx = 860.2^x$. Substituting 2 for x in this function yields $f2 = 860.2^2$, which is equivalent to $f2 = 860.04$, or $f2 = 3.44$. Either 3.44 or 86 / 25 may be entered as the correct answer.

Alternate approach: It's given that $f0 = 86$ and that for each increase in x by 1, the value of fx decreases by 80%. Therefore, when $x = 1$, the value of fx is $(100 - 80)\%$, or 20%, of 86, which can be expressed as 0.2086 . Since $0.2086 = 17.2$, the value of $f1$ is 17.2. Similarly, when $x = 2$, the value of fx is 20% of 17.2, which can be expressed as $0.2017.2$. Since $0.2017.2 = 3.44$, the value of $f2$ is 3.44. Either 3.44 or 86 / 25 may be entered as the correct answer.

Question Difficulty: Hard

Question ID dfbf8890

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: dfbf8890

A quadratic function models a projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. The model estimates that the projectile was launched from an initial height of **7** meters above the ground and reached a maximum height of **51.1** meters above the ground **3** seconds after the launch. How many seconds after the launch does the model estimate that the projectile will return to a height of **7** meters?

- A. **3**
- B. **6**
- C. **7**
- D. **9**

ID: dfbf8890 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that a quadratic function models the projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. It follows that an equation representing the model can be written in the form $fx = ax - h^2 + k$, where fx is the projectile's estimated height above the ground, in meters, x seconds after the launch, a is a constant, and k is the maximum height above the ground, in meters, the model estimates the projectile reached h seconds after the launch. It's given that the model estimates the projectile reached a maximum height of 51.1 meters above the ground 3 seconds after the launch. Therefore, $k = 51.1$ and $h = 3$. Substituting 51.1 for k and 3 for h in the equation $fx = ax - h^2 + k$ yields $fx = ax - 3^2 + 51.1$. It's also given that the model estimates that the projectile was launched from an initial height of 7 meters above the ground. Therefore, when $x = 0$, $fx = 7$. Substituting 0 for x and 7 for fx in the equation $fx = ax - 3^2 + 51.1$ yields $7 = a0 - 3^2 + 51.1$, or $7 = 9a + 51.1$. Subtracting 51.1 from both sides of this equation yields $-44.1 = 9a$. Dividing both sides of this equation by 9 yields $-4.9 = a$. Substituting -4.9 for a in the equation $fx = ax - 3^2 + 51.1$ yields $fx = -4.9x - 3^2 + 51.1$. Therefore, the equation $fx = -4.9x - 3^2 + 51.1$ models the projectile's height, in meters, above the ground x seconds after it was launched. The number of seconds after the launch that the model estimates that the projectile will return to a height of 7 meters is the value of x when $fx = 7$. Substituting 7 for fx in $fx = -4.9x - 3^2 + 51.1$ yields $7 = -4.9x - 3^2 + 51.1$. Subtracting 51.1 from both sides of this equation yields $-44.1 = -4.9x - 3^2$. Dividing both sides of this equation by -4.9 yields $9 = x - 3^2$. Taking the square root of both sides of this equation yields two equations: $3 = x - 3$ and $-3 = x - 3$. Adding 3 to both sides of the equation $3 = x - 3$ yields $6 = x$. Adding 3 to both sides of the equation $-3 = x - 3$ yields $0 = x$. Since 0 seconds after the launch represents the time at which the projectile was launched, 6 must be the number of seconds the model estimates that the projectile will return to a height of 7 meters.

Alternate approach: It's given that a quadratic function models the projectile's height, in meters, above the ground in terms of the time, in seconds, after it was launched. It's also given that the model estimates that the projectile was launched from an initial height of 7 meters above the ground and reached a maximum height of 51.1 meters above the ground 3 seconds after the launch. Since the model is quadratic, and quadratic functions are symmetric, the model estimates that for any given height less than the maximum height, the time the projectile takes to travel from the given height to the maximum height is the same as the time the

projectile takes to travel from the maximum height back to the given height. Thus, since the model estimates the projectile took 3 seconds to travel from 7 meters above the ground to its maximum height of 51.1 meters above the ground, the model also estimates the projectile will take 3 more seconds to travel from its maximum height of 51.1 meters above the ground back to 7 meters above the ground. Thus, the model estimates that the projectile will return to a height of 7 meters 3 seconds after it reaches its maximum height, which is 6 seconds after the launch.

Choice A is incorrect. The model estimates that 3 seconds after the launch, the projectile reached a height of 51.1 meters, not 7 meters.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID cdc398ca

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: cdc398ca

$$p(t) = 90,000(1.06)^t$$

The given function p models the population of Lowell t years after a census. Which of the following functions best models the population of Lowell m months after the census?

- A. $r(m) = \frac{90,000}{12}(1.06)^m$
- B. $r(m) = 90,000\left(\frac{1.06}{12}\right)^m$
- C. $r(m) = 90,000\left(\frac{1.06}{12}\right)^{\frac{m}{12}}$
- D. $r(m) = 90,000(1.06)^{\frac{m}{12}}$

ID: cdc398ca Answer

Correct Answer: D

Rationale

Choice D is correct. It’s given that the function p models the population of Lowell t years after a census. Since there are 12 months in a year, m months after the census is equivalent to $\frac{m}{12}$ years after the census. Substituting $\frac{m}{12}$ for t in the equation $pt = 90,0001.06^t$ yields $p\frac{m}{12} = 90,0001.06^{\frac{m}{12}}$. Therefore, the function r that best models the population of Lowell m months after the census is $rm = 90,0001.06^{\frac{m}{12}}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 583767ba

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 583767ba

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the xy -plane, the graph of $y = f(x) - 15$ has a y -intercept at $(0, -\frac{99}{7})$. The product of a and b is $\frac{65}{7}$. What is the value of a ?

ID: 583767ba Answer

Correct Answer: 5

Rationale

The correct answer is 5. It's given that $fx = -a^x + b$. Substituting $-a^x + b$ for fx in the equation $y = fx - 15$ yields $y = -a^x + b - 15$. It's given that the y -intercept of the graph of $y = fx - 15$ is 0, $-\frac{99}{7}$. Substituting 0 for x and $-\frac{99}{7}$ for y in the equation $y = -a^x + b - 15$ yields $-\frac{99}{7} = -a^0 + b - 15$, which is equivalent to $-\frac{99}{7} = -1 + b - 15$, or $-\frac{99}{7} = b - 16$. Adding 16 to both sides of this equation yields $\frac{13}{7} = b$. It's given that the product of a and b is $\frac{65}{7}$, or $ab = \frac{65}{7}$. Substituting $\frac{13}{7}$ for b in this equation yields $a\frac{13}{7} = \frac{65}{7}$. Dividing both sides of this equation by $\frac{13}{7}$ yields $a = 5$.

Question Difficulty: Hard

Question ID de3cc97b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: de3cc97b

The first term of a sequence is **9**. Each term after the first is **4** times the preceding term. If *w* represents the *n*th term of the sequence, which equation gives *w* in terms of *n*?

- A. $w = 4(9^n)$
- B. $w = 4(9^{n-1})$
- C. $w = 9(4^n)$
- D. $w = 9(4^{n-1})$

ID: de3cc97b Answer

Correct Answer: D

Rationale

Choice D is correct. Since *w* represents the *n*th term of the sequence and 9 is the first term of the sequence, the value of *w* is 9 when the value of *n* is 1. Since each term after the first is 4 times the preceding term, the value of *w* is 94 when the value of *n* is 2. Therefore, the value of *w* is 944, or 94^2 , when the value of *n* is 3. More generally, the value of *w* is 94^{n-1} for a given value of *n*. Therefore, the equation $w = 94^{n-1}$ gives *w* in terms of *n*.

Choice A is incorrect. This equation describes a sequence for which the first term is 36, rather than 9, and each term after the first is 9, rather than 4, times the preceding term.

Choice B is incorrect. This equation describes a sequence for which the first term is 4, rather than 9, and each term after the first is 9, rather than 4, times the preceding term.

Choice C is incorrect. This equation describes a sequence for which the first term is 36, rather than 9.

Question Difficulty: Hard

Question ID 42045837

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 42045837

A right rectangular prism has a height of **9** inches. The length of the prism's base is x inches, which is **7** inches more than the width of the prism's base. Which function V gives the volume of the prism, in cubic inches, in terms of the length of the prism's base?

- A. $V(x) = x(x + 9)(x + 7)$
- B. $V(x) = x(x + 9)(x - 7)$
- C. $V(x) = 9x(x + 7)$
- D. $V(x) = 9x(x - 7)$

ID: 42045837 Answer

Correct Answer: D

Rationale

Choice D is correct. The volume of a right rectangular prism can be represented by a function V that gives the volume of the prism, in cubic inches, in terms of the length of the prism's base. The volume of a right rectangular prism is equal to the area of its base times its height. It's given that the length of the prism's base is x inches, which is 7 inches more than the width of the prism's base. This means that the width of the prism's base is $x - 7$ inches. It follows that the area of the prism's base, in square inches, is $xx - 7$ and the volume, in cubic inches, of the prism is $xx - 79$. Thus, the function V that gives the volume of this right rectangular prism, in cubic inches, in terms of the length of the prism's base, x , is $Vx = 9xx - 7$.

Choice A is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base and the width of the base were 7 inches more than its length.

Choice B is incorrect. This function would give the volume of the prism if the height were 9 inches more than the length of its base.

Choice C is incorrect. This function would give the volume of the prism if the width of the base were 7 inches more than its length, rather than the length of the base being 7 inches more than its width.

Question Difficulty: Hard

Question ID ef091e05

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: ef091e05

$f(x) = \frac{a-19}{x} + 5$

In the given function f , a is a constant. The graph of function f in the xy -plane, where $y = f(x)$, is translated 3 units down and 4 units to the right to produce the graph of $y = g(x)$. Which equation defines function g ?

- A. $g(x) = \frac{a-19}{x+4} + 2$
- B. $g(x) = \frac{a-19}{x-4} + 2$
- C. $g(x) = \frac{a-22}{x+4} + 5$
- D. $g(x) = \frac{a-22}{x-4} + 5$

ID: ef091e05 Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the graph of $y = gx$ is produced by translating the graph of $y = fx$ 3 units down and 4 units to the right in the xy -plane. Therefore, function g can be defined by an equation in the form $gx = fx - 4 - 3$. Function f is defined by the equation $fx = \frac{a-19}{x} + 5$, where a is a constant. Substituting $x - 4$ for x in the equation $fx = \frac{a-19}{x} + 5$ yields $fx - 4 = \frac{a-19}{x-4} + 5$. Substituting $\frac{a-19}{x-4} + 5$ for $fx - 4$ in the equation $gx = fx - 4 - 3$ yields $gx = \frac{a-19}{x-4} + 5 - 3$, or $gx = \frac{a-19}{x-4} + 2$. Therefore, the equation that defines function g is $gx = \frac{a-19}{x-4} + 2$.

Choice A is incorrect. This equation defines a function whose graph is produced by translating the graph of $y = fx$ 3 units down and 4 units to the left, not 3 units down and 4 units to the right.

Choice C is incorrect. This equation defines a function whose graph is produced by translating the graph of $y = fx$ 4 units to the left, not 3 units down and 4 units to the right.

Choice D is incorrect. This equation defines a function whose graph is produced by translating the graph of $y = fx$ 4 units to the right, not 3 units down and 4 units to the right.

Question Difficulty: Hard

Question ID feba5a37

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: feba5a37

A rectangle has an area of **155** square inches. The length of the rectangle is **4** inches less than **7** times the width of the rectangle. What is the width of the rectangle, in inches?

ID: feba5a37 Answer

Correct Answer: 5

Rationale

The correct answer is 5. Let x represent the width, in inches, of the rectangle. It's given that the length of the rectangle is 4 inches less than 7 times its width, or $7x - 4$ inches. The area of a rectangle is equal to its width multiplied by its length. Multiplying the width, x inches, by the length, $7x - 4$ inches, yields $x(7x - 4)$ square inches. It's given that the rectangle has an area of 155 square inches, so it follows that $x(7x - 4) = 155$, or $7x^2 - 4x = 155$. Subtracting 155 from both sides of this equation yields $7x^2 - 4x - 155 = 0$. Factoring the left-hand side of this equation yields $(7x + 31)(x - 5) = 0$. Applying the zero product property to this equation yields two solutions: $x = -\frac{31}{7}$ and $x = 5$. Since x is the rectangle's width, in inches, which must be positive, the value of x is 5. Therefore, the width of the rectangle, in inches, is 5.

Question Difficulty: Hard

Question ID 429593eb

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 429593eb

x	y
21	-8
23	8
25	-8

The table shows three values of x and their corresponding values of y , where $y = f(x) + 4$ and f is a quadratic function. What is the y -coordinate of the y -intercept of the graph of $y = f(x)$ in the xy -plane?

ID: 429593eb Answer

Correct Answer: -2112

Rationale

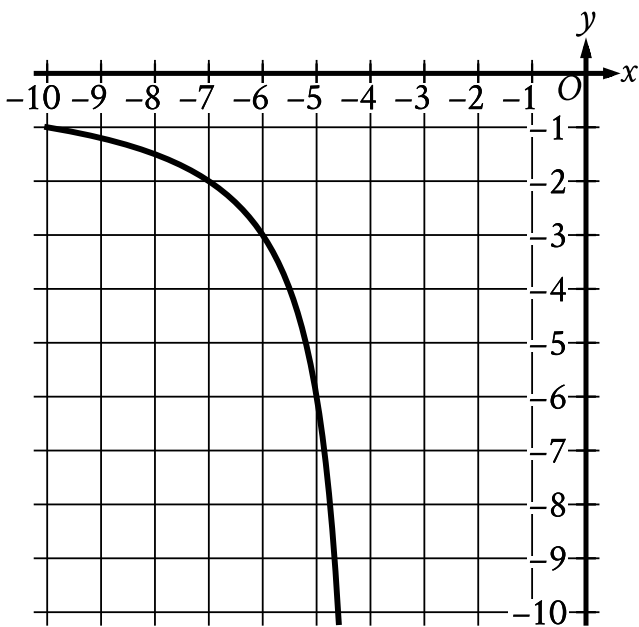
The correct answer is -2,112. It's given that f is a quadratic function. It follows that f can be defined by an equation of the form $fx = ax - h^2 + k$, where a , h , and k are constants. It's also given that the table shows three values of x and their corresponding values of y , where $y = fx + 4$. Substituting $ax - h^2 + k$ for fx in this equation yields $y = ax - h^2 + k + 4$. This equation represents a quadratic relationship between x and y , where $k + 4$ is either the maximum or the minimum value of y , which occurs when $x = h$. For quadratic relationships between x and y , the maximum or minimum value of y occurs at the value of x halfway between any two values of x that have the same corresponding value of y . The table shows that x -values of 21 and 25 correspond to the same y -value, -8. Since 23 is halfway between 21 and 25, the maximum or minimum value of y occurs at an x -value of 23. The table shows that when $x = 23$, $y = 8$. It follows that $h = 23$ and $k + 4 = 8$. Subtracting 4 from both sides of the equation $k + 4 = 8$ yields $k = 4$. Substituting 23 for h and 4 for k in the equation $y = ax - h^2 + k + 4$ yields $y = ax - 23^2 + 4 + 4$, or $y = ax - 23^2 + 8$. The value of a can be found by substituting any x -value and its corresponding y -value for x and y , respectively, in this equation. Substituting 25 for x and -8 for y in this equation yields $-8 = a25 - 23^2 + 8$, or $-8 = a2^2 + 8$. Subtracting 8 from both sides of this equation yields $-16 = a2^2$, or $-16 = 4a$. Dividing both sides of this equation by 4 yields $-4 = a$. Substituting -4 for a , 23 for h , and 4 for k in the equation $fx = ax - h^2 + k$ yields $fx = -4x - 23^2 + 4$. The y -intercept of the graph of $y = fx$ in the xy -plane is the point on the graph where $x = 0$. Substituting 0 for x in the equation $fx = -4x - 23^2 + 4$ yields $f0 = -40 - 23^2 + 4$, or $f0 = -4-23^2 + 4$. This is equivalent to $f0 = -2,112$, so the y -intercept of the graph of $y = fx$ in the xy -plane is 0, -2,112. Thus, the y -coordinate of the y -intercept of the graph of $y = fx$ in the xy -plane is -2,112.

Question Difficulty: Hard

Question ID cec444a7

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: cec444a7



The rational function f is defined by an equation in the form $f(x) = \frac{a}{x+b}$, where a and b are constants. The partial graph of $y = f(x)$ is shown. If $g(x) = f(x + 4)$, which equation could define function g ?

- A. $g(x) = \frac{6}{x}$
- B. $g(x) = \frac{6}{x+4}$
- C. $g(x) = \frac{6}{x+8}$
- D. $g(x) = \frac{6(x+4)}{x+4}$

ID: cec444a7 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that $fx = \frac{a}{x+b}$ and that the graph shown is a partial graph of $y = fx$. Substituting y for fx in the equation $fx = \frac{a}{x+b}$ yields $y = \frac{a}{x+b}$. The graph passes through the point -7, -2. Substituting -7 for x and -2 for y in the equation $y = \frac{a}{x+b}$ yields $-2 = \frac{a}{-7+b}$. Multiplying each side of this equation by $-7 + b$ yields $-2 \cdot -7 + b = a$, or $14 - 2b = a$. The graph also passes through the point -5, -6. Substituting -5 for x and -6 for y in the equation $y = \frac{a}{x+b}$ yields $-6 = \frac{a}{-5+b}$. Multiplying each side of this equation by $-5 + b$ yields $-6 \cdot -5 + b = a$, or $30 - 6b = a$. Substituting $14 - 2b$ for a in this equation yields $30 - 6b = 14 - 2b$. Adding $6b$ to each side of this equation yields $30 = 14 + 4b$. Subtracting 14 from each side of this equation yields $16 = 4b$. Dividing each side of this equation by 4 yields $4 = b$. Substituting 4 for b in the equation $14 - 2b = a$ yields $14 - 2 \cdot 4 = a$, or $6 = a$.

Substituting 6 for a and 4 for b in the equation $fx = \frac{a}{x+b}$ yields $fx = \frac{6}{x+4}$. It's given that $gx = fx + 4$. Substituting $x + 4$ for x in the equation $fx = \frac{6}{x+4}$ yields $fx + 4 = \frac{6}{x+4+4}$, which is equivalent to $fx + 4 = \frac{6}{x+8}$. It follows that $gx = \frac{6}{x+8}$.

Choice A is incorrect. This could define function g if $gx = fx - 4$.

Choice B is incorrect. This could define function g if $gx = fx$.

Choice D is incorrect. This could define function g if $gx = fx \cdot x + 4$.

Question Difficulty: Hard

Question ID 3b0ffb71

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 3b0ffb71

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the xy -plane, the graph of $y = f(x) - 12$ has a y -intercept at $(0, -\frac{75}{7})$. The product of a and b is $\frac{320}{7}$. What is the value of a ?

ID: 3b0ffb71 Answer

Correct Answer: 20

Rationale

The correct answer is 20. It's given that $fx = -a^x + b$. Substituting $-a^x + b$ for fx in the equation $y = fx - 12$ yields $y = -a^x + b - 12$. It's given that the y -intercept of the graph of $y = fx - 12$ is $0, -\frac{75}{7}$. Substituting 0 for x and $-\frac{75}{7}$ for y in the equation $y = -a^x + b - 12$ yields $-\frac{75}{7} = -a^0 + b - 12$, which is equivalent to $-\frac{75}{7} = -1 + b - 12$, or $-\frac{75}{7} = b - 13$. Adding 13 to both sides of this equation yields $\frac{16}{7} = b$. It's given that the product of a and b is $\frac{320}{7}$, or $ab = \frac{320}{7}$. Substituting $\frac{16}{7}$ for b in this equation yields $a\frac{16}{7} = \frac{320}{7}$. Dividing both sides of this equation by $\frac{16}{7}$ yields $a = 20$.

Question Difficulty: Hard

Question ID 55087e8b

Assessment	Test	Domain	Skill	Difficulty
PSAT/NMSQT & PSAT 10	Math	Advanced Math	Nonlinear functions	

ID: 55087e8b

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object has an initial height of **10** feet above the ground and reaches its maximum height of **1,034** feet above the ground **8** seconds after being launched. Based on the model, what is the height, in feet, of the object above the ground **10** seconds after being launched?

- A. **234**
- B. **778**
- C. **970**
- D. **1,014**

ID: 55087e8b Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that a quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. This quadratic function can be defined by an equation of the form $fx = ax - h^2 + k$, where fx is the height of the object x seconds after it was launched, and a , h , and k are constants such that the function reaches its maximum value, k , when $x = h$. Since the model indicates the object reaches its maximum height of 1,034 feet above the ground 8 seconds after being launched, fx reaches its maximum value, 1,034, when $x = 8$. Therefore, $k = 1,034$ and $h = 8$. Substituting 8 for h and 1,034 for k in the function $fx = ax - h^2 + k$ yields $fx = ax - 8^2 + 1,034$. Since the model indicates the object has an initial height of 10 feet above the ground, the value of fx is 10 when $x = 0$. Substituting 0 for x and 10 for fx in the equation $fx = ax - 8^2 + 1,034$ yields $10 = a0 - 8^2 + 1,034$, or $10 = 64a + 1,034$. Subtracting 1,034 from both sides of this equation yields $64a = -1,024$. Dividing both sides of this equation by 64 yields $a = -16$. Therefore, the model can be represented by the equation $fx = -16x - 8^2 + 1,034$. Substituting 10 for x in this equation yields $f10 = -1610 - 8^2 + 1,034$, or $f10 = 970$. Therefore, based on the model, 10 seconds after being launched, the height of the object above the ground is 970 feet.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard