Math Test – No Calculator Answer Explanations

Question 1

KEY: D

DIFFICULTY: Medium

No Calculator

Choice D is correct. The expression $15x + 24ax$ contains two terms with common factors. One of the common factors is x. Factoring x from the expression gives $x(15 + 24a)$, which can also be written as $(15 + 24a)x$.

Choices A, B, and C are incorrect and may result from incorrectly combining and/or factoring the two terms of the expression. One can check that the expressions in each of these choices are not equivalent to the given expression. For example, in choice A, for $x = 1$ and $a = 0$, the value of the given expression is 15 and the value of the expression $39ax^2$ is 0.

Question 2

KEY: A

DIFFICULTY: Easy

No Calculator

Choice A is correct. Dividing each side of the equation $d = rt$ by t results in an equation that expresses r in terms of the other variables: $r = \frac{d}{t}$.

Choices B, C, and D are incorrect and may result from algebraic errors when rewriting the given equation.

Question 3

KEY: B

DIFFICULTY: Medium

No Calculator

Choice B is correct. The equation $x = y - 4$ can be rewritten as $y = x + 4$. Substituting $x + 4$ for y in the other equation gives $x + 4 = x^2 + 3x - 4$, which can be rewritten as $x^2 + 2x - 8 = 0$. Since -4 and 2 are the two numbers whose sum is -2 and whose product is -8, they are the solutions to the equation $x^2 + 2x - 8 = 0$. From the equation $y = x + 4$, it follows that the solutions of the system are $(-4, 0)$ and $(2, 6)$. Therefore, of the given choices, $(2, 6)$ is the correct answer.

Choices A and C are incorrect because each of these ordered pairs satisfies the quadratic equation but not the linear equation. Choice D is incorrect because this ordered pair satisfies the linear equation but not the quadratic equation.

Question 4

KEY: C

DIFFICULTY: Medium

No Calculator

Choice C is correct. The given equation can be rewritten as $x^2 - 4x + 3 = 0$. Since 1 and 3 are two numbers whose sum is 4 and whose product is 3, it follows that they are the solutions to the equation $x^2 - 4x + 3 = 0$. Therefore, of the choices given, only 3 can be a solution to the original equation.

Choices A, B, and D are incorrect because none of these values satisfy the given equation.

Question 5

KEY: C

DIFFICULTY: Medium

No Calculator

Choice C is correct. Multiplying each side of the second equation by 3 and then adding the equations eliminates x, as follows:

$$\begin{cases} -3x - 4y = 20 \\ 3x - 30y = 48 \end{cases}$$
$$0 - 34y = 68$$

Solving the obtained equation for y gives $y = -2$.

Substituting -2 for y in the second equation of the system gives $x - 10(-2) = 16$, which simplifies to $x + 20 = 16$, or $x = -4$.

Choices A, B, and D are incorrect because there is no solution to the system for which the x-coordinate is one of the numbers given in these choices. For example, substituting -14 for x in the second equation gives $y = -3$. But the pair $(-14, -3)$ does not satisfy the first equation, and it is therefore not a solution to the system of equations.

Question 6

KEY: B

DIFFICULTY: Medium

No Calculator

Choice B is correct. If the equation $y = 36 + 18x$ is graphed in the xy-plane, the y-intercept is at $(0, 36)$. Since y represents the height, in inches, of a typical apple tree and x represents the number of years after it was planted, it follows that the number 36 represents the height, in inches, of a typical apple tree when $x = 0$; that is, the height, in inches, at the time the apple tree is planted.

Choice A is incorrect and may be the result of confusing the age of the tree with its height. Choice C is incorrect because the equation provided does not indicate when a typical apple tree will stop growing. Choice D is incorrect and may be the result of confusing the y-intercept with the slope of the line $y = 36 + 18x$.

Question 7

KEY: A

DIFFICULTY: Medium

No Calculator

Choice A is correct. The cost, in dollars, of Giovanni's 2 shirts is $19.40 \times 2 = 38.80$, and the cost, in dollars, of his p sweaters is $24.80 \times p = 24.80p$. Additionally, he paid an 8% sales tax. To include the sales tax in the total cost, the combined cost of shirts and sweaters should be multiplied by 1.08. Therefore, the total cost, in dollars, of Giovanni's purchases, y, can be expressed as $1.08(38.80 + 24.80p)$.

Choice B is incorrect and may result from using the factor $1 - 0.08 = 0.92$, instead of $1 + 0.08 = 1.08$, to calculate the sales

tax and from multiplying by this factor on the wrong side of the equation. Choice C is incorrect and may result from multiplying by the sales tax factor on the wrong side of the equation. Choice D is incorrect and may result from using the factor $1 - 0.08 = 0.92$ instead of $1 + 0.08 = 1.08$ to calculate the sales tax.

Question 8

Choice B is correct. Any line that passes through the point $(-3, 3)$ and has a positive slope will intersect the y-axis at a point $(0, b)$ with $b > 3$; that is, such a line will have a y-intercept greater than 3. Therefore, a line that has a positive slope and a negative y-intercept cannot pass through the point $(-3, 3)$.

KEY: B

DIFFICULTY: Medium

No Calculator

Choices A, C, and D are incorrect because they are points that a line with a positive slope and a negative y-intercept could pass through. For example, in choice A, the line with equation $y = \frac{1}{3}x - 2$ has a positive slope $\left(\frac{1}{3}\right)$ and a negative y-intercept (-2) but passes through the point $(-3, -3)$.

Question 9

Choice C is correct. If the length, in centimeters, of one piece of rope is represented by q, and each piece of rope must be at least 270 centimeters and no more than 280 centimeters long, then it follows that $270 \le q \le 280$. In turn, the total length x, in centimeters, of rope needed for the parachute is $18q$ because 18 pieces are needed. So, since $x = 18q$, multiplying all the terms of the inequality $270 \le q \le 280$ by 18 gives $(270 \times 18) \le 18q \le (280 \times 18)$, or $4{,}860 \le x \le 5{,}040$.

KEY: C

DIFFICULTY: Medium

No Calculator

Choice A is incorrect and may result from mistakenly using x for the length, in centimeters, of one piece of rope instead of the total length of rope. Choice B is incorrect and may result from multiplying the single-piece lower limit for length by 18 and then adding 10 to create the total upper limit, instead of multiplying both the single-piece lower and upper limits by 18. Choice D is incorrect and may result from multiplying the single-piece upper limit for length by 18 and then subtracting 10 to create the total lower limit, instead of multiplying both the single-piece lower and upper limits by 18.

Question 10

Choice D is correct. Since the carpenter needs to buy both nails and screws, at least one box of each needs to be purchased. This can be expressed by the pair of inequalities $n \ge 1$ and $s \ge 1$. However, the number of boxes the carpenter can buy is limited by a budget of \$60. The amount, in dollars, the carpenter spends on nails or screws can

KEY: D

DIFFICULTY: Medium

No Calculator

be expressed as the price of each box multiplied by the number of each type of box, which is $12.99n$ for nails and $14.99s$ for screws. And since this total cannot exceed \$60, it follows that $12.99n + 14.99s \leq 60$.

Choice A is incorrect because the first inequality allows the total cost of nails and screws to exceed the carpenter's budget of \$60, and the second inequality incorrectly expresses the constraint on the number of boxes that the carpenter can buy. That number must be greater than 1, since the carpenter must buy at least one box of nails and one box of screws. Choice B is incorrect because the second equation incorrectly expresses the constraint on the number of boxes that the carpenter can buy. That number must be greater than 1, since the carpenter must buy at least one box of nails and one box of screws. Choice C is incorrect because the first inequality allows for the total cost to exceed the carpenter's budget of \$60.

Question 11

KEY: A

DIFFICULTY: Hard

No Calculator

Choice A is correct. In the figure, triangles ABC and BDC are similar because each has an angle that measures 28°, and they share angle C. Thus their corresponding sides are in proportion. The sides AB in triangle ABC and BD in triangle BDC correspond to each other because they are opposite the same angle in both triangles (angle C), and the sides BC in triangle ABC and DC in triangle BDC correspond to each other because they are opposite the congruent angles with measure 28° in the corresponding triangles. Therefore, $\frac{AB}{BC} = \frac{BD}{DC}$.

Choices B, C, and D are incorrect because they are ratios that do not have the same value as $\frac{AB}{BC}$ and are likely the result of misunderstanding which triangles are similar or which sides of the triangles are corresponding sides.

Question 12

KEY: C

DIFFICULTY: Hard

No Calculator

Choice C is correct. After distributing the outside exponents to each expression within the parentheses by the rules of exponents, the left side of the equation can be rewritten as

$$\left(x^2y^3\right)^{\frac{1}{2}}\left(x^2y^3\right)^{\frac{1}{3}} = \left(x^{(2)\left(\frac{1}{2}\right)}y^{(3)\left(\frac{1}{2}\right)}\right)\left(x^{(2)\left(\frac{1}{3}\right)}y^{(3)\left(\frac{1}{3}\right)}\right) = \left(xy^{\frac{3}{2}}\right)\left(x^{\frac{2}{3}}y\right).$$

Multiplying the expressions within the parentheses and applying the exponent rules yields $x^{1+\frac{2}{3}}y^{\frac{3}{2}+1} = x^{\frac{5}{3}}y^{\frac{5}{2}}$, which means the equation $x^{\frac{5}{3}}y^{\frac{5}{2}} = x^{\frac{a}{3}}y^{\frac{a}{2}}$ is true for all positive values of x and y. It

follows that the corresponding exponents of x and y on both sides of the equation must be equal, which yields $a = 5$.

Choices A, B, and D are incorrect and may result from errors when applying the rules of exponents to the given expression.

Question 13

KEY: B

DIFFICULTY: Hard

No Calculator

Choice B is correct. The graph of $y = (x - 6)(x + 12)$ is a parabola that opens upward and has a vertical axis of symmetry. The vertex of the parabola lies on this axis of symmetry, and the x-intercepts of the parabola are equidistant from the axis of symmetry. Since the equation $y = (x - 6)(x + 12)$ is in factored form, the x-intercepts of its graph are $(6, 0)$ and $(-12, 0)$. Therefore, the axis of symmetry is the line $x = \frac{6 + (-12)}{2}$, or $x = -3$. Because the vertex lies on the line $x = -3$, the x-coordinate of the vertex must also be $x = -3$.

Choices A, C, and D are incorrect and may result from misunderstanding the relationship between the given equation and the x-intercepts of the parabola as well as the relationship between the x-intercepts of the parabola and the x-coordinate of the parabola's vertex. For example, choice C may result from mistakenly taking the x-intercepts of the graph of $y = (x - 6)(x + 12)$ as $(-6, 0)$ and $(12, 0)$ instead of as $(6, 0)$ and $(-12, 0)$.

Question 14

KEY: 2

DIFFICULTY: Medium

No Calculator

The correct answer is 2. If a linear equation is written in the form $mx + n = px + r$, where $m = p$ and $n = r$, then the linear equation is satisfied by *any* value of x and will have infinitely many solutions. Distributing 7 on the right-hand side of the given equation yields $21x + 14 = 21x + 7a$. Therefore, the equation will have infinitely many solutions if $14 = 7a$; that is, if $a = 2$.

Question 15

KEY: 90

DIFFICULTY: Hard

No Calculator

The correct answer is 90. Juliene practiced twice as long on Monday as she did on Tuesday. Therefore, if x is the number of minutes Juliene practiced on Tuesday, then $2x$ is the number of minutes she practiced on Monday. The total amount of time Juliene practiced on the two days is 2 hours and 15 minutes, which is equal to 135 minutes. Thus, the equation $x + 2x = 135$ must be true. This simplifies to $3x = 135$, and so $x = 45$. The number of minutes Juliene practiced on Monday is $2x$, which is equal to $2x = 2(45) = 90$.

Question 16

The correct answer is 1. It is given that one factor of the quadratic expression is $3x + 4$. Thus, $12x^2 + ax - 20 = (3x + 4)(mx + p)$, where a, m, and p are integers. Multiplying out the right-hand side of the equation gives $12x^2 + ax - 20 = 3mx^2 + (3p + 4m)x + 4p$. It follows that $12 = 3m$, $a = 3p + 4m$, and $-20 = 4p$. Dividing both sides of the equation $12 = 3m$ by 3 gives $m = 4$. Dividing both sides of the equation $-20 = 4p$ by 4 gives $p = -5$. Finally, substituting $m = 4$ and $p = -5$ in the equation $a = 3p + 4m$ gives $a = 3(-5) + 4(4) = 1$.

KEY: 1

DIFFICULTY: Hard

No Calculator

Question 17

The correct answer is 0. Multiplying out the given expression yields $(ax + by)(cx - dy) = acx^2 + (bc - ad)xy - bdy^2$. Since $ad = bc$, the coefficient of the xy term, $bc - ad$, is 0.

KEY: 0

DIFFICULTY: Hard

No Calculator