Question ID a4c7e214

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a4c7e214

$$x = 8a(b + 9)$$

The given equation relates the positive numbers a, b, and x. Which equation correctly expresses a in terms of b and x?

A. $a = \frac{x}{8} - (b + 9)$

B. $a = \frac{x}{8(b+9)}$

C. $a = \frac{8(b+9)}{x}$

D. $a = 8x(b + 9)$

ID: a4c7e214 Answer

Correct Answer: B

Rationale

Choice B is correct. To express a in terms of b and x, the given equation can be rewritten such that a is isolated on one side of the equation. Since it's given that b is a positive number, $b + 9$ is not equal to zero. Therefore, dividing both sides of the given equation by $8b + 9$ yields the equivalent equation $\frac{x}{8b + 9} = a$, or $a = \frac{x}{8b + 9}$.

Choice A is incorrect. This equation is equivalent to $x = 8a + b + 9$.

Choice C is incorrect. This equation is equivalent to $x = \frac{8b + 9}{a}$.

Choice D is incorrect. This equation is equivalent to $x = \frac{a}{8b + 9}$.

Question Difficulty: Hard

Question ID d8afa04f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: d8afa04f

$$5x^2 - 37x - 24 = 0$$

What is the positive solution to the given equation?

A. $\frac{3}{5}$

B. 3

C. 8

D. 37

ID: d8afa04f Answer

Correct Answer: C

Rationale

Choice C is correct. The left-hand side of the given equation can be factored as $5x + 3x - 8$. Therefore, the given equation, $5x^2 - 37x - 24 = 0$, can be written as $5x + 3x - 8 = 0$. Applying the zero product property to this equation yields $5x + 3 = 0$ and $x - 8 = 0$. Subtracting 3 from both sides of the equation $5x + 3 = 0$ yields $5x = -3$. Dividing both sides of this equation by 5 yields $x = -\frac{3}{5}$. Adding 8 to both sides of the equation $x - 8 = 0$ yields $x = 8$. Therefore, the two solutions to the given equation, $5x^2 - 37x - 24 = 0$, are $-\frac{3}{5}$ and 8. It follows that 8 is the positive solution to the given equation.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d5884aab

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: d5884aab

$$5(x+7)=15(x-17)(x+7)$$

What is the sum of the solutions to the given equation?

ID: d5884aab Answer

Correct Answer: 10.33, 31/3

Rationale

The correct answer is $\frac{31}{3}$. Subtracting $5x+7$ from each side of the given equation yields $0=15x$ - $17x+7$ - $5x+7$. Since $5x+7$ is a common factor of each of the terms on the right-hand side of this equation, it can be rewritten as $0=5x+73x$ - 17 - 1. This is equivalent to $0=5x+73x$ - 51 - 1, or $0=5x+73x$ - 52. Dividing both sides of this equation by 5 yields $0=x+73x$ - 52. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either $x+7=0$ or $3x$ - $52=0$. Subtracting 7 from both sides of the equation $x+7=0$ yields $x=$ -7. Adding 52 to both sides of the equation $3x$ - $52=0$ yields $3x=52$. Dividing both sides of this equation by 3 yields $x=\frac{52}{3}$. Therefore, the solutions to the given equation are -7 and $\frac{52}{3}$. It follows that the sum of the solutions to the given equation is -7 + $\frac{52}{3}$, which is equivalent to -$\frac{21}{3}+\frac{52}{3}$, or $\frac{31}{3}$. Note that 31/3 and 10.33 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID a505a103

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a505a103

$$y = 18$$
$$y = -3(x - 18)^2 + 15$$

If the given equations are graphed in the *xy*-plane, at how many points do the graphs of the equations intersect?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: a505a103 Answer

Correct Answer: D

Rationale

Choice D is correct. A point x, y is a solution to a system of equations if it lies on the graphs of both equations in the *xy*-plane. In other words, a solution to a system of equations is a point x, y at which the graphs intersect. It's given that the first equation is $y = 18$. Substituting 18 for y in the second equation yields $18 = -3x - 18^2 + 15$. Subtracting 15 from each side of this equation yields $3 = -3x - 18^2$. Dividing each side of this equation by -3 yields $-1 = x - 18^2$. Since the square of a real number is at least 0, this equation can't have any real solutions. Therefore, the graphs of the equations intersect at zero points.

Alternate approach: The graph of the second equation is a parabola that opens downward and has a vertex at $18, 15$. Therefore, the maximum value of this parabola occurs when $y = 15$. The graph of the first equation is a horizontal line at 18 on the *y*-axis, or $y = 18$. Since 18 is greater than 15, or the horizontal line is above the vertex of the parabola, the graphs of these equations intersect at zero points.

Choice A is incorrect. The graph of $y = 15$, not $y = 18$, and the graph of the second equation intersect at exactly one point.

Choice B is incorrect. The graph of any horizontal line such that the value of y is less than 15, not greater than 15, and the graph of the second equation intersect at exactly two points.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID de064610

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: de064610

$$b - 72 = \frac{x}{y}$$

The given equation relates the positive numbers b, x, and y. Which equation correctly expresses x in terms of b and y?

A. $x = \frac{b-72}{y}$

B. $x = by - 72$

C. $x = \frac{by-72}{y}$

D. $x = by - 72y$

ID: de064610 Answer

Correct Answer: D

Rationale

Choice D is correct. Multiplying both sides of the given equation by y yields $y\left(b - 72\right) = x$. Distributing on the left-hand side of this equation yields $by - 72y = x$, or $x = by - 72y$. Therefore, the equation $x = by - 72y$ correctly expresses x in terms of b and y.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID cb0a29df

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: cb0a29df

$$7m = 5(n + p)$$

The given equation relates the positive numbers m, n, and p. Which equation correctly gives n in terms of m and p?

A. $n = \frac{5p}{7m}$

B. $n = \frac{7m}{5} - p$

C. $n = 5(7m) + p$

D. $n = 7m - 5 - p$

ID: cb0a29df Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the equation $7m = 5n + p$ relates the positive numbers m, n, and p. Dividing both sides of the given equation by 5 yields $\frac{7m}{5} = n + p$. Subtracting p from both sides of this equation yields $\frac{7m}{5} - p = n$, or $n = \frac{7m}{5} - p$. It follows that the equation $n = \frac{7m}{5} - p$ correctly gives n in terms of m and p.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fc135182

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: fc135182

$$|x - 9| + 45 = 63$$

What is the sum of the solutions to the given equation?

ID: fc135182 Answer

Correct Answer: 18

Rationale

The correct answer is 18. Subtracting 45 from each side of the given equation yields x - 9 = 18. By the definition of absolute value, if x - 9 = 18, then x - 9 = 18 or x - 9 = - 18. Adding 9 to each side of the equation x - 9 = 18 yields x = 27. Adding 9 to each side of the equation x - 9 = - 18 yields x = - 9. Therefore, the solutions to the given equation are 27 and -9, and it follows that the sum of the solutions to the given equation is 27 + -9, or 18.

Question Difficulty: Hard

Question ID b3abe641

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: b3abe641

The solutions to $x^2 + 6x + 7 = 0$ are r and s, where $r < s$. The solutions to $x^2 + 8x + 8 = 0$ are t and u, where $t < u$. The solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. What is the value of c?

ID: b3abe641 Answer

Correct Answer: 31

Rationale

The correct answer is 31. Subtracting 7 from both sides of the equation $x^2 + 6x + 7 = 0$ yields $x^2 + 6x = -7$. To complete the square, adding $\frac{6}{2}^2$, or 3^2, to both sides of this equation yields $x^2 + 6x + 3^2 = -7 + 3^2$, or $x + 3^2 = 2$. Taking the square root of both sides of this equation yields $x + 3 = \pm\sqrt{2}$. Subtracting 3 from both sides of this equation yields $x = -3 \pm \sqrt{2}$. Therefore, the solutions r and s to the equation $x^2 + 6x + 7 = 0$ are $-3 - \sqrt{2}$ and $-3 + \sqrt{2}$. Since $r < s$, it follows that $r = -3 - \sqrt{2}$ and $s = -3 + \sqrt{2}$. Subtracting 8 from both sides of the equation $x^2 + 8x + 8 = 0$ yields $x^2 + 8x = -8$. To complete the square, adding $\frac{8}{2}^2$, or 4^2, to both sides of this equation yields $x^2 + 8x + 4^2 = -8 + 4^2$, or $x + 4^2 = 8$. Taking the square root of both sides of this equation yields $x + 4 = \pm\sqrt{8}$, or $x + 4 = \pm 2\sqrt{2}$. Subtracting 4 from both sides of this equation yields $x = -4 \pm 2\sqrt{2}$. Therefore, the solutions t and u to the equation $x^2 + 8x + 8 = 0$ are $-4 - 2\sqrt{2}$ and $-4 + 2\sqrt{2}$. Since $t < u$, it follows that $t = -4 - 2\sqrt{2}$ and $u = -4 + 2\sqrt{2}$. It's given that the solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. It follows that this equation can be written as $x - r + tx - s + u = 0$, which is equivalent to $x^2 - r + t + s + ux + r + ts + u = 0$. Therefore, the value of c is $r + ts + u$. Substituting $-3 - \sqrt{2}$ for r, $-4 - 2\sqrt{2}$ for t, $-3 + \sqrt{2}$ for s, and $-4 + 2\sqrt{2}$ for u in this equation yields $-3 - \sqrt{2} + -4 - 2\sqrt{2} - 3 + \sqrt{2} + -4 + 2\sqrt{2}$, which is equivalent to $-7 - 3\sqrt{2} - 7 + 3\sqrt{2}$, or $-7 - 7 - 3\sqrt{2}\,3\sqrt{2}$, which is equivalent to $49 - 18$, or 31. Therefore, the value of c is 31.

Question Difficulty: Hard

Question ID 31305921

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 31305921

$$8x + y = -11$$
$$2x^2 = y + 341$$

The graphs of the equations in the given system of equations intersect at the point (x, y) in the xy-plane. What is a possible value of x?

A. -15

B. -11

C. 2

D. 8

ID: 31305921 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graphs of the equations in the given system of equations intersect at the point x, y. Therefore, this intersection point is a solution to the given system. The solution can be found by isolating y in each equation. The given equation $8x + y =$ - 11 can be rewritten to isolate y by subtracting $8x$ from both sides of the equation, which gives $y =$ - $8x$ - 11. The given equation $2x^2 = y + 341$ can be rewritten to isolate y by subtracting 341 from both sides of the equation, which gives $2x^2$ - 341 $= y$. With each equation solved for y, the value of y from one equation can be substituted into the other, which gives $2x^2$ - 341 $=$ - $8x$ - 11. Adding $8x$ and 11 to both sides of this equation results in $2x^2 + 8x$ - 330 $= 0$. Dividing both sides of this equation by 2 results in $x^2 + 4x$ - 165 $= 0$. This equation can be rewritten by factoring the left-hand side, which yields $x + 15x$ - 11 $= 0$. By the zero-product property, if $x + 15x$ - 11 $= 0$, then $x + 15 = 0$, or x - 11 $= 0$. It follows that $x =$ - 15, or $x = 11$. Since only -15 is given as a choice, a possible value of x is -15.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 31368b54

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 31368b54

$$y + k = x + 26$$
$$y - k = x^2 - 5x$$

In the given system of equations, k is a constant. The system has exactly one distinct real solution. What is the value of k?

ID: 31368b54 Answer

Correct Answer: 17.5, 35/2

Rationale

The correct answer is $\frac{35}{2}$. Subtracting the second equation from the first equation yields $y + k$ - y - $k = x + 26$ - x^2 - $5x$, or $2k =$ - $x^2 + 6x + 26$. This is equivalent to x^2 - $6x + 2k$ - $26 = 0$. It's given that the system has exactly one distinct real solution; therefore, this equation has exactly one distinct real solution. An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one distinct real solution when the discriminant, b^2 - $4ac$, is equal to 0. The equation x^2 - $6x + \left(2k - 26\right) = 0$ is of this form, where $a = 1$, $b =$ - 6, and $c = 2k$ - 26. Substituting these values into the discriminant, b^2 - $4ac$, yields -6^2 - $412k$ - 26. Setting the discriminant equal to 0 yields -6^2 - $412k$ - $26 = 0$, or -$8k + 140 = 0$. Subtracting 140 from both sides of this equation yields -$8k =$ - 140. Dividing both sides of this equation by -8 yields $k = \frac{35}{2}$. Note that 35/2 and 17.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID cb1a6c01

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: cb1a6c01

$$P = N(19 - C)$$

The given equation relates the positive numbers P, N, and C. Which equation correctly expresses C in terms of P and N?

A. $C = \frac{19+P}{N}$

B. $C = \frac{19-P}{N}$

C. $C = 19 + \frac{P}{N}$

D. $C = 19 - \frac{P}{N}$

ID: cb1a6c01 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the values of P, N, and C are positive. Therefore, dividing each side of the given equation by N yields $\frac{P}{N} = 19 - C$. Subtracting 19 from each side of this equation yields $\frac{P}{N} - 19 = -C$. Dividing each side of this equation by -1 yields $19 - \frac{P}{N} = C$, or $C = 19 - \frac{P}{N}$.

Choice A is incorrect. This equation is equivalent to $P = NC - 19$, not $P = N19 - C$.

Choice B is incorrect. This equation is equivalent to $P = 19 - NC$, not $P = N19 - C$.

Choice C is incorrect. This equation is equivalent to $P = NC - 19$, not $P = N19 - C$.

Question Difficulty: Hard

Question ID d42a332a

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: d42a332a

$$(5x + 4)(2x - 5) = 0$$

Which of the following is a solution to the given equation?

A. $-\frac{5}{2}$

B. $-\frac{5}{4}$

C. $-\frac{4}{5}$

D. $-\frac{2}{5}$

ID: d42a332a Answer

Correct Answer: C

Rationale

Choice C is correct. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either $5x + 4 = 0$ or $2x - 5 = 0$. Subtracting 4 from each side of the equation $5x + 4 = 0$ yields $5x = -4$. Dividing each side of this equation by 5 yields $x = -\frac{4}{5}$. Adding 5 to each side of the equation $2x - 5 = 0$ yields $2x = 5$. Dividing each side of this equation by 2 yields $x = \frac{5}{2}$. It follows that the solutions to the given equation are $-\frac{4}{5}$ and $\frac{5}{2}$. Therefore, $-\frac{4}{5}$ is a solution to the given equation.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b6b7302e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: b6b7302e

$$y = x^2 + 14x + 48$$
$$x + 8 = 11$$

The solution to the given system of equations is (x, y). What is the value of y?

ID: b6b7302e Answer

Correct Answer: 99

Rationale

The correct answer is 99. In the given system of equations, the second equation is $x + 8 = 11$. Subtracting 8 from both sides of this equation yields $x = 3$. In the given system of equations, the first equation is $y = x^2 + 14x + 48$. Substituting 3 for x in this equation yields $y = 3^2 + 143 + 48$, or $y = 99$. Therefore, the solution to the given system of equations is $x, y = 3, 99$. Thus, the value of y is 99.

Question Difficulty: Hard

Question ID 79e90fe0

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 79e90fe0

$$|-5x + 13| = 73$$

What is the sum of the solutions to the given equation?

A. $-\frac{146}{5}$

B. -12

C. 0

D. $\frac{26}{5}$

ID: 79e90fe0 Answer

Correct Answer: D

Rationale

Choice D is correct. By the definition of absolute value, if $-5x + 13 = 73$, then $-5x + 13 = 73$ or $-5x + 13 = -73$. Subtracting 13 from both sides of the equation $-5x + 13 = 73$ yields $-5x = 60$. Dividing both sides of this equation by -5 yields $x = -12$. Subtracting 13 from both sides of the equation $-5x + 13 = -73$ yields $-5x = -86$. Dividing both sides of this equation by -5 yields $x = \frac{86}{5}$. Therefore, the solutions to the given equation are -12 and $\frac{86}{5}$, and it follows that the sum of the solutions to the given equation is $-12 + \frac{86}{5}$, or $\frac{26}{5}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect. This is a solution, not the sum of the solutions, to the given equation.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2053d7ae

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 2053d7ae

$$z^2 + 10z - 24 = 0$$

What is one of the solutions to the given equation?

ID: 2053d7ae Answer

Correct Answer: 2, -12

Rationale

The correct answer is either 2 or -12. The left-hand side of the given equation can be rewritten by factoring. The two values that multiply to -24 and add to 10 are 12 and -2. It follows that the given equation can be rewritten as $z + 12z - 2 = 0$. Setting each factor equal to 0 yields two equations: $z + 12 = 0$ and $z - 2 = 0$. Subtracting 12 from both sides of the equation $z + 12 = 0$ results in $z = -12$. Adding 2 to both sides of the equation $z - 2 = 0$ results in $z = 2$. Note that 2 and -12 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 0b733fe3

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 0b733fe3

$$7x^2 - 20x - 32 = 0$$

What is the positive solution to the given equation?

ID: 0b733fe3 Answer

Correct Answer: 4

Rationale

The correct answer is 4. The left-hand side of the given equation can be factored as $7x + 8x$ - 4. Therefore, the given equation, $7x^2$ - $20x$ - $32 = 0$, can be written as $7x + 8x$ - $4 = 0$. Applying the zero product property to this equation yields $7x + 8 = 0$ and x - $4 = 0$. Subtracting 8 from both sides of the equation $7x + 8 = 0$ yields $7x =$ -8. Dividing both sides of this equation by 7 yields $x = -\frac{8}{7}$. Adding 4 to both sides of the equation x - $4 = 0$ yields $x = 4$. Therefore, the two solutions to the given equation, $7x^2$ - $20x$ - $32 = 0$, are $-\frac{8}{7}$ and 4. It follows that 4 is the positive solution to the given equation.

Question Difficulty: Hard

Question ID 8fac5670

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 8fac5670

$$x^2 - 12x + 27 = 0$$

How many distinct real solutions does the given equation have?

A. Exactly two

B. Exactly one

C. Zero

D. Infinitely many

ID: 8fac5670 Answer

Correct Answer: A

Rationale

Choice A is correct. The number of solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be determined by the value of the discriminant, b^2 - $4ac$. If the value of the discriminant is positive, then the quadratic equation has exactly two distinct real solutions. If the value of the discriminant is equal to zero, then the quadratic equation has exactly one real solution. If the value of the discriminant is negative, then the quadratic equation has zero real solutions. In the given equation, x^2 - $12x + 27 = 0$, $a = 1$, $b =$ -12, and $c = 27$. Substituting these values for a, b, and c in b^2 - $4ac$ yields -12^2 - 4127, or 36. Since the value of its discriminant is positive, the given equation has exactly two distinct real solutions.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID f16b622c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: f16b622c

$$\frac{14x}{7y} = 2\sqrt{w + 19}$$

The given equation relates the distinct positive real numbers w, x, and y. Which equation correctly expresses w in terms of x and y?

A. $w = \sqrt{\frac{x}{y}} - 19$

B. $w = \sqrt{\frac{28x}{14y}} - 19$

C. $w = \left(\frac{x}{y}\right)^2 - 19$

D. $w = \left(\frac{28x}{14y}\right)^2 - 19$

ID: f16b622c Answer

Correct Answer: C

Rationale

Choice C is correct. Dividing each side of the given equation by 2 yields $\frac{14x}{14y} = \frac{2\sqrt{w + 19}}{2}$, or $\frac{x}{y} = \sqrt{w + 19}$. Because it's given that each of the variables is positive, squaring each side of this equation yields the equivalent equation $\frac{x^2}{y} = w + 19$. Subtracting 19 from each side of this equation yields $\frac{x^2}{y}$ - 19 = w, or $w = \frac{x^2}{y}$ - 19.

Choice A is incorrect. This equation isn't equivalent to the given equation.

Choice B is incorrect. This equation isn't equivalent to the given equation.

Choice D is incorrect. This equation isn't equivalent to the given equation.

Question Difficulty: Hard

Question ID 882e1392

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 882e1392

$$v = -\frac{w}{150x}$$

The given equation relates the distinct positive numbers v, w, and x. Which equation correctly expresses w in terms of v and x?

A. $w = -150vx$

B. $w = -\frac{150v}{x}$

C. $w = -\frac{x}{150v}$

D. $w = v + 150x$

ID: 882e1392 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that x is positive. Therefore, multiplying each side of the given equation by $-150x$ yields $-150xv = w$, which is equivalent to $w = -150vx$. Thus, the equation $w = -150vx$ correctly expresses w in terms of v and x.

Choice B is incorrect. This equation is equivalent to $v = -\frac{wx}{150}$.

Choice C is incorrect. This equation is equivalent to $v = -\frac{x}{150w}$.

Choice D is incorrect. This equation is equivalent to $v = w - 150x$.

Question Difficulty: Hard

Question ID a0923004

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a0923004

$$2x^2 - 8x - 7 = 0$$

One solution to the given equation can be written as $\frac{8-\sqrt{k}}{4}$, where k is a constant. What is the value of k?

ID: a0923004 Answer

Correct Answer: 120

Rationale

The correct answer is 120. The solutions to a quadratic equation of the form $ax^2 + bx + c = 0$ can be calculated using the quadratic formula and are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The given equation is in the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -8$, and $c = -7$. It follows that the solutions to the given equation are $x = \frac{8 \pm \sqrt{-8^2 - 42 \cdot -7}}{22}$, which is equivalent to $x = \frac{8 \pm \sqrt{64 + 56}}{4}$, or $x = \frac{8 \pm \sqrt{120}}{4}$. It's given that one solution to the equation $2x^2 - 8x - 7 = 0$ can be written as $\frac{8 - \sqrt{k}}{4}$. The solution $\frac{8 - \sqrt{120}}{4}$ is in this form. Therefore, the value of k is 120.

Question Difficulty: Hard

Question ID e65a96bc

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e65a96bc

$$x^2 - 2x - 9 = 0$$

One solution to the given equation can be written as $1 + \sqrt{k}$, where k is a constant. What is the value of k?

A. 8

B. 10

C. 20

D. 40

ID: e65a96bc Answer

Correct Answer: B

Rationale

Choice B is correct. Adding 9 to each side of the given equation yields $x^2 - 2x = 9$. To complete the square, adding 1 to each side of this equation yields $x^2 - 2x + 1 = 9 + 1$, or $(x - 1)^2 = 10$. Taking the square root of each side of this equation yields $x - 1 = \pm\sqrt{10}$. Adding 1 to each side of this equation yields $x = 1 \pm \sqrt{10}$. Since it's given that one of the solutions to the equation can be written as $1 + \sqrt{k}$, the value of k must be 10.

Alternate approach: It's given that $1 + \sqrt{k}$ is a solution to the given equation. It follows that $x = 1 + \sqrt{k}$. Substituting $1 + \sqrt{k}$ for x in the given equation yields $(1 + \sqrt{k})^2 - 2(1 + \sqrt{k}) - 9 = 0$, or $(1 + \sqrt{k})(1 + \sqrt{k}) - 2(1 + \sqrt{k}) - 9 = 0$. Expanding the products on the left-hand side of this equation yields $1 + 2\sqrt{k} + k - 2 - 2\sqrt{k} - 9 = 0$, or $k - 10 = 0$. Adding 10 to each side of this equation yields $k = 10$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 7bbcc18f

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 7bbcc18f

$$p = \frac{k}{4j+9}$$

The given equation relates the distinct positive numbers p, k, and j. Which equation correctly expresses $4j + 9$ in terms of p and k?

A. $4j + 9 = \frac{k}{p}$

B. $4j + 9 = kp$

C. $4j + 9 = k - p$

D. $4j + 9 = \frac{p}{k}$

ID: 7bbcc18f Answer

Correct Answer: A

Rationale

Choice A is correct. To express $4j + 9$ in terms of p and k, the given equation must be solved for $4j + 9$. Since it's given that j is a positive number, $4j + 9$ is not equal to zero. Therefore, multiplying both sides of the given equation by $4j + 9$ yields the equivalent equation $p4j + 9 = k$. Since it's given that p is a positive number, p is not equal to zero. Therefore, dividing each side of the equation $p4j + 9 = k$ by p yields the equivalent equation $4j + 9 = \frac{k}{p}$.

Choice B is incorrect. This equation is equivalent to $p = \frac{4j + 9}{k}$.

Choice C is incorrect. This equation is equivalent to $p = k$ - $4j$ - 9.

Choice D is incorrect. This equation is equivalent to $p = k4j + 9$.

Question Difficulty: Hard

Question ID 42a24450

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 42a24450

$$64x^2 - (16a + 4b)x + ab = 0$$

In the given equation, a and b are positive constants. The sum of the solutions to the given equation is $k(4a + b)$, where k is a constant. What is the value of k?

ID: 42a24450 Answer

Correct Answer: .0625, 1/16

Rationale

The correct answer is $\frac{1}{16}$. Let p and q represent the solutions to the given equation. Then, the given equation can be rewritten as $64x$ - px - $q = 0$, or $64x^2$ - $64p + q + pq = 0$. Since this equation is equivalent to the given equation, it follows that -$16a + 4b =$ - $64p + q$. Dividing both sides of this equation by -64 yields $\frac{16a + 4b}{64} = p + q$, or $\frac{1}{16}4a + b = p + q$. Therefore, the sum of the solutions to the given equation, $p + q$, is equal to $\frac{1}{16}4a + b$. Since it's given that the sum of the solutions to the given equation is $k4a + b$, where k is a constant, it follows that $k = \frac{1}{16}$. Note that 1/16, .0625, 0.062, and 0.063 are examples of ways to enter a correct answer.

Alternate approach: The given equation can be rewritten as $64x^2$ - $44a + bx + ab = 0$, where a and b are positive constants. Dividing both sides of this equation by 4 yields $16x^2$ - $4a + bx + \frac{ab}{4} = 0$. The solutions for a quadratic equation in the form $Ax^2 + Bx + C = 0$, where A, B, and C are constants, can be calculated using the quadratic formula, $x = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$ and $x = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$. It follows that the sum of the solutions to a quadratic equation in the form $Ax^2 + Bx + C = 0$ is $\frac{-B + \sqrt{B^2 - 4AC}}{2A} + \frac{-B - \sqrt{B^2 - 4AC}}{2A}$, which can be rewritten as $\frac{-B + -B + \sqrt{B^2 - 4AC} - \sqrt{B^2 - 4AC}}{2A}$, which is equivalent to $\frac{-2B}{2A}$, or $-\frac{B}{A}$. In the equation $16x^2$ - $4a + bx + \frac{ab}{4} = 0$, $A = 16$, $B =$ - $4a + b$, and $C = \frac{ab}{4}$. Substituting 16 for A and -$4a + b$ for B in $-\frac{B}{A}$ yields -$\frac{-4a + b}{16}$, which can be rewritten as $\frac{1}{16}4a + b$. Thus, the sum of the solutions to the given equation is $\frac{1}{16}4a + b$. Since it's given that the sum of the solutions to the given equation is $k4a + b$, where k is a constant, it follows that $k = \frac{1}{16}$.

Question Difficulty: Hard

Question ID af94bc02

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: af94bc02

$$\frac{(x+9)(x-9)}{x+9} = 7$$

What is the solution to the given equation?

A. 7

B. 9

C. 16

D. 63

ID: af94bc02 Answer

Correct Answer: C

Rationale

Choice C is correct. Since the left-hand side of the given equation has a factor of $x + 9$ in both the numerator and the denominator, the solution to the given equation can be found by solving the equation $x - 9 = 7$. Adding 9 to both sides of this equation yields $x = 16$. Substituting 16 for x in the given equation yields $\frac{16 + 9}{16 + 9} = 7$, or $7 = 7$. Therefore, the solution to the given equation is 16.

Choice A is incorrect. Substituting 7 for x in the given equation yields $\frac{7 + 9}{7 + 9} = 7$, or $-2 = 7$, which is false.

Choice B is incorrect. Substituting 9 for x in the given equation yields $\frac{9 + 9}{9 + 9} = 7$, or $0 = 7$, which is false.

Choice D is incorrect. Substituting 63 for x in the given equation yields $\frac{63 + 9}{63 + 9} = 7$, or $54 = 7$, which is false.

Question Difficulty: Hard

Question ID 1ad081fd

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 1ad081fd

If $4\sqrt{2x} = 16$, what is the value of $6x$?

A. 24

B. 48

C. 72

D. 96

ID: 1ad081fd Answer

Correct Answer: B

Rationale

Choice B is correct. Dividing each side of the given equation by 4 yields $\sqrt{2x} = 4$. Squaring both sides of this equation yields $2x = 16$. Multiplying each side of this equation by 3 yields $6x = 48$. Therefore, the value of $6x$ is 48.

Choice A is incorrect. This is the value of $3x$, not $6x$.

Choice C is incorrect. This is the value of $9x$, not $6x$.

Choice D is incorrect. This is the value of $12x$, not $6x$.

Question Difficulty: Hard

Question ID 7b2f17ba

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 7b2f17ba

$$x^2 = -841$$

How many distinct real solutions does the given equation have?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 7b2f17ba Answer

Correct Answer: D

Rationale

Choice D is correct. Since the square of a real number is never negative, the given equation isn't true for any real value of x. Therefore, the given equation has zero distinct real solutions.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 04e3c183

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 04e3c183

$$y = 4x$$
$$y = x^2 - 12$$

A solution to the given system of equations is (x, y), where $x > 0$. What is the value of x?

ID: 04e3c183 Answer

Correct Answer: 6

Rationale

The correct answer is 6. It's given that $y = 4x$ and $y = x^2$ - 12. Since $y = 4x$, substituting $4x$ for y in the second equation of the given system yields $4x = x^2$ - 12. Subtracting $4x$ from both sides of this equation yields $0 = x^2$ - $4x$ - 12. This equation can be rewritten as $0 = x$ - $6x + 2$. By the zero product property, x - $6 = 0$ or $x + 2 = 0$. Adding 6 to both sides of the equation x - $6 = 0$ yields $x = 6$. Subtracting 2 from both sides of the equation $x + 2 = 0$ yields $x =$ - 2. Therefore, solutions to the given system of equations occur when $x = 6$ and when $x =$ - 2. It's given that a solution to the given system of equations is x, y, where $x > 0$. Since 6 is greater than 0, it follows that the value of x is 6.

Question Difficulty: Hard

Question ID fd76bbbb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: fd76bbbb

$$38x^2 = 38(9)$$

What is the negative solution to the given equation?

ID: fd76bbbb Answer

Correct Answer: -3

Rationale

The correct answer is -3. Dividing both sides of the given equation by 38 yields $x^2 = 9$. Taking the square root of both sides of this equation yields the solutions $x = 3$ and $x = -3$. Therefore, the negative solution to the given equation is -3.

Question Difficulty: Hard

Question ID e45c168b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e45c168b

$$-2x^2 + 20x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

A. -68

B. -50

C. -32

D. 0

ID: e45c168b Answer

Correct Answer: B

Rationale

Choice B is correct. It's given that the equation $-2x^2 + 20x + c = 0$, where c is a constant, has exactly one solution. A quadratic equation of the form $ax^2 + bx + c = 0$ has exactly one solution if and only if its discriminant, $b^2 - 4ac$, is equal to zero. It follows that for the given equation, $a = -2$ and $b = 20$. Substituting -2 for a and 20 for b in $b^2 - 4ac$ yields $20^2 - 4(-2)(c)$, or $400 + 8c$. Since the discriminant must equal zero, it follows that $400 + 8c = 0$. Subtracting 400 from both sides of this equation yields $8c = -400$. Dividing each side of this equation by 8 yields $c = -50$. Therefore, the value of c is -50.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 17c92e06

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 17c92e06

$$w^2 + 12w - 40 = 0$$

Which of the following is a solution to the given equation?

A. $6 - 2\sqrt{19}$

B. $2\sqrt{19}$

C. $\sqrt{19}$

D. $-6 + 2\sqrt{19}$

ID: 17c92e06 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding 40 to both sides of the given equation yields $w^2 + 12w = 40$. To complete the square, adding $\frac{12}{2}^2$, or 6^2, to both sides of this equation yields $w^2 + 12w + 6^2 = 40 + 6^2$, or $w + 6^2 = 76$. Taking the square root of both sides of this equation yields $w + 6 = \pm\sqrt{76}$, or $w + 6 = \pm 2\sqrt{19}$. Subtracting 6 from both sides of this equation yields $w = -6 \pm 2\sqrt{19}$. Therefore, the solutions to the given equation are $-6 + 2\sqrt{19}$ and $-6 - 2\sqrt{19}$. Of these two solutions, only $-6 + 2\sqrt{19}$ is given as a choice.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID e4f33b6e

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e4f33b6e

In the *xy*-plane, a line with equation $2y = c$ for some constant c intersects a parabola at exactly one point. If the parabola has equation $y = -2x^2 + 9x$, what is the value of c?

ID: e4f33b6e Answer

Correct Answer: 20.25, 81/4

Rationale

The correct answer is $\frac{81}{4}$. The given linear equation is $2y = c$. Dividing both sides of this equation by 2 yields $y = \frac{c}{2}$. Substituting $\frac{c}{2}$ for y in the equation of the parabola yields $\frac{c}{2} = -2x^2 + 9x$. Adding $2x^2$ and $-9x$ to both sides of this equation yields $2x^2 - 9x + \frac{c}{2} = 0$. Since it's given that the line and the parabola intersect at exactly one point, the equation $2x^2 - 9x + \frac{c}{2} = 0$ must have exactly one solution. An equation of the form $Ax^2 + Bx + C = 0$, where A, B, and C are constants, has exactly one solution when the discriminant, $B^2 - 4AC$, is equal to 0. In the equation $2x^2 - 9x + \frac{c}{2} = 0$, where $A = 2$, $B = -9$, and $C = \frac{c}{2}$, the discriminant is $-9^2 - 42\frac{c}{2}$. Setting the discriminant equal to 0 yields $-9^2 - 42\frac{c}{2} = 0$, or $81 - 4c = 0$. Adding $4c$ to both sides of this equation yields $81 = 4c$. Dividing both sides of this equation by 4 yields $c = \frac{81}{4}$. Note that 81/4 and 20.25 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID a678d2b8

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a678d2b8

In the *xy*-plane, the graph of the equation $y = -x^2 + 9x - 100$ intersects the line $y = c$ at exactly one point. What is the value of c?

A. $-\frac{481}{4}$

B. -100

C. $-\frac{319}{4}$

D. $-\frac{9}{2}$

ID: a678d2b8 Answer

Correct Answer: C

Rationale

Choice C is correct. In the *xy*-plane, the graph of the line $y = c$ is a horizontal line that crosses the *y*-axis at $y = c$ and the graph of the quadratic equation $y = -x^2 + 9x - 100$ is a parabola. A parabola can intersect a horizontal line at exactly one point only at its vertex. Therefore, the value of c should be equal to the *y*-coordinate of the vertex of the graph of the given equation. For a quadratic equation in vertex form, $y = a(x - h)^2 + k$, the vertex of its graph in the *xy*-plane is (h, k). The given quadratic equation, $y = -x^2 + 9x - 100$, can be rewritten as $y = -x^2 - 2\frac{9}{2}x + \frac{9}{2}^2 + \frac{9}{2}^2 - 100$, or $y = -(x - \frac{9}{2})^2 + -\frac{319}{4}$. Thus, the value of c is equal to $-\frac{319}{4}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d4f08434

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: d4f08434

$$x^2 - 40x - 10 = 0$$

What is the sum of the solutions to the given equation?

A. 0

B. 5

C. 10

D. 40

ID: d4f08434 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding 10 to each side of the given equation yields $x^2 - 40x = 10$. To complete the square, adding $\frac{40}{2}^2$, or 20^2, to each side of this equation yields $x^2 - 40x + 20^2 = 10 + 20^2$, or $x - 20^2 = 410$. Taking the square root of each side of this equation yields $x - 20 = \pm\sqrt{410}$. Adding 20 to each side of this equation yields $x = 20 \pm \sqrt{410}$. Therefore, the solutions to the given equation are $x = 20 + \sqrt{410}$ and $x = 20 - \sqrt{410}$. The sum of these solutions is $20 + \sqrt{410} + 20 - \sqrt{410}$, or 40.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID db0b6fc5

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: db0b6fc5

$$y = (x - 2)(x + 4)$$
$$y = 6x - 12$$

Which ordered pair (x, y) is the solution to the given system of equations?

A. $(0, 2)$

B. $(-4, 2)$

C. $(2, 0)$

D. $(2, -4)$

ID: db0b6fc5 Answer

Correct Answer: C

Rationale

Choice C is correct. The second equation in the given system of equations is $y = 6x - 12$. Substituting $6x - 12$ for y in the first equation of the given system yields $6x - 12 = x - 2x + 4$. Factoring 6 out of the left-hand side of this equation yields $6x - 2 = x - 2x + 4$. An expression with a factor of the form $x - a$ is equal to zero when $x = a$. Each side of this equation has a factor of $x - 2$, so each side of the equation is equal to zero when $x = 2$. Substituting 2 for x into the equation $6x - 2 = x - 2x + 4$ yields $62 - 2 = 2 - 22 + 4$, or $0 = 0$, which is true. Substituting 2 for x into the second equation in the given system of equations yields $y = 62 - 12$, or $y = 0$. Therefore, the solution to the system of equations is the ordered pair $2, 0$.

Choice A is incorrect and may result from switching the order of the solutions for x and y.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 316b2564

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 316b2564

$$\frac{12}{n} - \frac{2}{t} = -\frac{2}{w}$$

The given equation relates the variables n, t, and w, where $n > 0$, $t > 0$, and $w > t$. Which expression is equivalent to n?

A. $12tw$

B. $6(t - w)$

C. $\frac{w-t}{6tw}$

D. $\frac{6tw}{w-t}$

ID: 316b2564 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding $\frac{2}{t}$ to each side of the given equation yields $\frac{12}{n} = -\frac{2}{w} + \frac{2}{t}$. The fractions on the right side of this equation have a common denominator of tw; therefore, the equation can be written as $\frac{12}{n} = \frac{2w}{tw} - \frac{2t}{tw}$, or $\frac{12}{n} = \frac{2w - 2t}{tw}$, which is equivalent to $\frac{12}{n} = \frac{2w - t}{tw}$. Dividing each side of this equation by 2 yields $\frac{6}{n} = \frac{w - t}{tw}$. Since n, t, w, and $w - t$ are all positive quantities, taking the reciprocal of each side of the equation $\frac{6}{n} = \frac{w - t}{tw}$ yields an equivalent equation: $\frac{n}{6} = \frac{tw}{w - t}$. Multiplying each side of this equation by 6 yields $n = \frac{6tw}{w - t}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is equivalent to $\frac{1}{n}$ rather than n.

Question Difficulty: Hard

Question ID 67e0e5fb

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 67e0e5fb

$$x^2 - 5x - 24 = 0$$

What is the sum of the solutions to the given equation?

ID: 67e0e5fb Answer

Correct Answer: 5

Rationale

The correct answer is 5. The given quadratic equation can be rewritten in factored form as $(x - 8)(x + 3) = 0$. Based on the zero product property, it follows that $x - 8 = 0$ or $x + 3 = 0$. Adding 8 to both sides of the equation $x - 8 = 0$ yields $x = 8$. Subtracting 3 from both sides of the equation $x + 3 = 0$ yields $x = -3$. Therefore, the solutions to the given equation are 8 and -3. It follows that the sum of the solutions to the given equation is $8 + (-3)$, or 5.

Question Difficulty: Hard

Question ID 9445ac1c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 9445ac1c

$$y = 2x^2 - 21x + 64$$
$$y = 3x + a$$

In the given system of equations, a is a constant. The graphs of the equations in the given system intersect at exactly one point, (x, y), in the *xy*-plane. What is the value of x?

A. -8

B. -6

C. 6

D. 8

ID: 9445ac1c Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the graphs of the equations in the given system intersect at exactly one point, x, y, in the *xy*-plane. Therefore, x, y is the only solution to the given system of equations. The given system of equations can be solved by subtracting the second equation, $y = 3x + a$, from the first equation, $y = 2x^2 - 21x + 64$. This yields $y - y = 2x^2 - 21x + 64 - 3x + a$, or $0 = 2x^2 - 24x + 64 - a$. Since the given system has only one solution, this equation has only one solution. A quadratic equation in the form $rx^2 + sx + t = 0$, where r, s, and t are constants, has one solution if and only if the discriminant, $s^2 - 4rt$, is equal to zero. Substituting 2 for r, -24 for s, and $-a + 64$ for t in the expression $s^2 - 4rt$ yields $-24^2 - 4264 - a$. Setting this expression equal to zero yields $-24^2 - 4264 - a = 0$, or $8a + 64 = 0$. Subtracting 64 from both sides of this equation yields $8a = -64$. Dividing both sides of this equation by 8 yields $a = -8$. Substituting -8 for a in the equation $0 = 2x^2 - 24x + 64 - a$ yields $0 = 2x^2 - 24x + 64 + 8$, or $0 = 2x^2 - 24x + 72$. Factoring 2 from the right-hand side of this equation yields $0 = 2x^2 - 12x + 36$. Dividing both sides of this equation by 2 yields $0 = x^2 - 12x + 36$, which is equivalent to $0 = x - 6x - 6$, or $0 = x - 6^2$. Taking the square root of both sides of this equation yields $0 = x - 6$. Adding 6 to both sides of this equation yields $x = 6$.

Choice A is incorrect. This is the value of a, not x.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID a4178bb7

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a4178bb7

$$\frac{1}{7b} = \frac{11x}{y}$$

The given equation relates the positive numbers b, x, and y. Which equation correctly expresses x in terms of b and y?

A. $x = \frac{7by}{11}$

B. $x = y - 77b$

C. $x = \frac{y}{77b}$

D. $x = 77by$

ID: a4178bb7 Answer

Correct Answer: C

Rationale

Choice C is correct. Multiplying each side of the given equation by y yields the equivalent equation $\frac{y}{7b} = 11x$. Dividing each side of this equation by 11 yields $\frac{y}{77b} = x$, or $x = \frac{y}{77b}$.

Choice A is incorrect. This equation is not equivalent to the given equation.

Choice B is incorrect. This equation is not equivalent to the given equation.

Choice D is incorrect. This equation is not equivalent to the given equation.

Question Difficulty: Hard

Question ID 6902b109

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 6902b109

$$y = -2.5$$
$$y = x^2 + 8x + k$$

In the given system of equations, k is a positive integer constant. The system has no real solutions. What is the least possible value of k?

ID: 6902b109 Answer

Correct Answer: 14

Rationale

The correct answer is 14. It's given by the first equation of the system of equations that $y = -2.5$. Substituting -2.5 for y in the second given equation, $y = x^2 + 8x + k$, yields $-2.5 = x^2 + 8x + k$. Adding 2.5 to both sides of this equation yields $0 = x^2 + 8x + k + 2.5$. A quadratic equation of the form $0 = ax^2 + bx + c$, where a, b, and c are constants, has no real solutions if and only if its discriminant, $b^2 - 4ac$, is negative. In the equation $0 = x^2 + 8x + k + 2.5$, where k is a positive integer constant, $a = 1$, $b = 8$, and $c = k + 2.5$. Substituting 1 for a, 8 for b, and $k + 2.5$ for c in $b^2 - 4ac$ yields $8^2 - 41k + 2.5$, or $64 - 4k + 2.5$. Since this value must be negative, $64 - 4k + 2.5 < 0$. Adding $4k + 2.5$ to both sides of this inequality yields $64 < 4k + 2.5$. Dividing both sides of this inequality by 4 yields $16 < k + 2.5$. Subtracting 2.5 from both sides of this inequality yields $13.5 < k$. Since k is a positive integer constant, the least possible value of k is 14.

Question Difficulty: Hard

Question ID 1cab3587

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 1cab3587

$$(x - 47)^2 = 1$$

What is the sum of the solutions to the given equation?

ID: 1cab3587 Answer

Correct Answer: 94

Rationale

The correct answer is 94. Taking the square root of each side of the given equation yields $x - 47 = 1$ or $x - 47 = -1$. Adding 47 to both sides of the equation $x - 47 = 1$ yields $x = 48$. Adding 47 to both sides of the equation $x - 47 = -1$ yields $x = 46$. Therefore, the sum of the solutions to the given equation is $48 + 46$, or 94.

Question Difficulty: Hard

Question ID e5de1695

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e5de1695

$$x - 29 = (x - a)(x - 29)$$

Which of the following are solutions to the given equation, where a is a constant and $a > 30$?

I. a

II. $a + 1$

III. 29

A. I and II only

B. I and III only

C. II and III only

D. I, II, and III

ID: e5de1695 Answer

Correct Answer: C

Rationale

Choice C is correct. Subtracting the expression x - 29 from both sides of the given equation yields $0 = x$ - ax - 29 - x - 29, which can be rewritten as $0 = x$ - ax - 29 + -1x - 29. Since the two terms on the right-hand side of this equation have a common factor of x - 29, it can be rewritten as $0 = x$ - 29x - a + -1, or $0 = x$ - 29x - a - 1. Since x - a - 1 is equivalent to x - a + 1, the equation $0 = x$ - 29x - a - 1 can be rewritten as $0 = x$ - 29x - a + 1. By the zero product property, it follows that x - 29 = 0 or x - a + 1 = 0. Adding 29 to both sides of the equation x - 29 = 0 yields $x = 29$. Adding $a + 1$ to both sides of the equation x - a + 1 = 0 yields $x = a + 1$. Therefore, the two solutions to the given equation are 29 and $a + 1$. Thus, only $a + 1$ and 29, not a, are solutions to the given equation.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 34395f35

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 34395f35

$$14j + 5k = m$$

The given equation relates the numbers j, k, and m. Which equation correctly expresses k in terms of j and m?

A. $k = \frac{m-14j}{5}$

B. $k = \frac{1}{5}m - 14j$

C. $k = \frac{14j-m}{5}$

D. $k = 5m - 14j$

ID: 34395f35 Answer

Correct Answer: A

Rationale

Choice A is correct. Subtracting $14j$ from each side of the given equation results in $5k = m - 14j$. Dividing each side of this equation by 5 results in $k = \frac{m - 14j}{5}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ccb8cab1

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: ccb8cab1

In the xy-plane, a line with equation $2y = 4.5$ intersects a parabola at exactly one point. If the parabola has equation $y = -4x^2 + bx$, where b is a positive constant, what is the value of b?

ID: ccb8cab1 Answer

Correct Answer: 6

Rationale

The correct answer is 6. It's given that a line with equation $2y = 4.5$ intersects a parabola with equation $y = -4x^2 + bx$, where b is a positive constant, at exactly one point in the *xy*-plane. It follows that the system of equations consisting of $2y = 4.5$ and $y = -4x^2 + bx$ has exactly one solution. Dividing both sides of the equation of the line by 2 yields $y = 2.25$. Substituting 2.25 for y in the equation of the parabola yields $2.25 = -4x^2 + bx$. Adding $4x^2$ and subtracting bx from both sides of this equation yields $4x^2 - bx + 2.25 = 0$. A quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when the discriminant, $b^2 - 4ac$, is equal to zero. Substituting 4 for a and 2.25 for c in the expression $b^2 - 4ac$ and setting this expression equal to 0 yields $b^2 - 4(4)(2.25) = 0$, or $b^2 - 36 = 0$. Adding 36 to each side of this equation yields $b^2 = 36$. Taking the square root of each side of this equation yields $b = \pm 6$. It's given that b is positive, so the value of b is 6.

Question Difficulty: Hard

Question ID 900c3f03

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 900c3f03

$$p = 20 + \frac{16}{n}$$

The given equation relates the numbers p and n, where n is not equal to 0 and $p > 20$. Which equation correctly expresses n in terms of p?

A. $n = \frac{p-20}{16}$

B. $n = \frac{p}{16} + 20$

C. $n = \frac{p}{16} - 20$

D. $n = \frac{16}{p-20}$

ID: 900c3f03 Answer

Correct Answer: D

Rationale

Choice D is correct. To express n in terms of p, the given equation must be solved for n. Subtracting 9 from both sides of the given equation yields $p - 9 = \frac{14}{n}$. Since n is not equal to 0, multiplying both sides of this equation by n yields $p - 9n = 14$. It's given that $p > 9$, which means $p - 9$ is not equal to 0. Therefore, dividing both sides of $p - 9n = 14$ by $p - 9$ yields $\frac{p - 9n}{p - 9} = \frac{14}{p - 9}$, or $n = \frac{14}{p - 9}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 4c8b75ab

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 4c8b75ab

$$3x(x-4)(x+5)=0$$

What is one of the solutions to the given equation?

A. -4

B. 0

C. 3

D. 5

ID: 4c8b75ab Answer

Correct Answer: B

Rationale

Choice B is correct. Applying the zero product property to the given equation yields $3x = 0$, x - $4 = 0$, and $x + 5 = 0$. Dividing each side of the equation $3x = 0$ by 3 yields $x = 0$. Adding 4 to each side of the equation x - $4 = 0$ yields $x = 4$. Subtracting 5 from each side of the equation $x + 5 = 0$ yields x = -5. Therefore, the solutions to the given equation are 0, 4, and -5. Thus, one of the solutions to the given equation is 0.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 0a69855a

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 0a69855a

The equation $12t + b = c$ relates the variables t, b, and c. Which of the following correctly expresses the value of $c - b$ in terms of t?

A. $\frac{t}{12}$

B. t

C. $t + \frac{1}{12}$

D. $12t$

ID: 0a69855a Answer

Correct Answer: D

Rationale

Choice D is correct. Subtracting b from each side of the given equation yields $12t = c$ - b. Therefore, the expression $12t$ correctly expresses the value of c - b in terms of t.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID e00ebccc

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e00ebccc

$$y = x + 9$$
$$y = x^2 + 16x + 63$$

A solution to the given system of equations is (x, y). What is the greatest possible value of x?

A. -6

B. 7

C. 9

D. 63

ID: e00ebccc Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $y = x + 9$ and $y = x^2 + 16x + 63$; therefore, it follows that $x + 9 = x^2 + 16x + 63$. This equation can be rewritten as $x + 9 = x + 9x + 7$. Subtracting $x + 9$ from both sides of this equation yields $0 = x + 9x + 7 - x + 9$. This equation can be rewritten as $0 = x + 9x + 7 - 1$, or $0 = x + 9x + 6$. By the zero product property, $x + 9 = 0$ or $x + 6 = 0$. Subtracting 9 from both sides of the equation $x + 9 = 0$ yields $x = -9$. Subtracting 6 from both sides of the equation $x + 6 = 0$ yields $x = -6$. Therefore, the given system of equations has solutions, x, y, that occur when $x = -9$ and $x = -6$. Since -6 is greater than -9, the greatest possible value of x is -6.

Choice B is incorrect. This is the negative of the greatest possible value of x when $y = 0$ for the second equation in the given system of equations.

Choice C is incorrect. This is the value of y when $x = 0$ for the first equation in the given system of equations.

Choice D is incorrect. This is the value of y when $x = 0$ for the second equation in the given system of equations.

Question Difficulty: Hard

Question ID 5b85333b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 5b85333b

If $\frac{42}{x} = 7x$, what is the value of $7x^2$?

A. 6

B. 7

C. 42

D. 294

ID: 5b85333b Answer

Correct Answer: C

Rationale

Choice C is correct. Multiplying both sides of the given equation by x yields $42 = 7x^2$. Therefore, the value of $7x^2$ is 42.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID d0cb954c

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: d0cb954c

$$x(x+1)-56=4x(x-7)$$

What is the sum of the solutions to the given equation?

ID: d0cb954c Answer

Correct Answer: 29/3, 9.666, 9.667

Rationale

The correct answer is $\frac{29}{3}$. Applying the distributive property to the left-hand side of the given equation, $xx + 1$ - 56, yields $x^2 + x$ - 56. Applying the distributive property to the right-hand side of the given equation, $4xx$ - 7, yields $4x^2$ - $28x$. Thus, the equation becomes $x^2 + x$ - $56 = 4x^2$ - $28x$. Combining like terms on the left- and right-hand sides of this equation yields $0 = 4x^2$ - x^2 + -28x - $x + 56$, or $3x^2$ - $29x + 56 = 0$. For a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, the quadratic formula gives the solutions to the equation in the form $x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$. Substituting 3 for a, -29 for b, and 56 for c from the equation $3x^2$ - $29x + 56 = 0$ into the quadratic formula yields $x = \frac{29 \pm \sqrt{\text{-}29^2 - 4356}}{23}$, or $x = \frac{29}{6} \pm \frac{13}{6}$. It follows that the solutions to the given equation are $\frac{29}{6} + \frac{13}{6}$ and $\frac{29}{6}$ - $\frac{13}{6}$. Adding these two solutions gives the sum of the solutions: $\frac{29}{6} + \frac{13}{6} + \frac{29}{6}$ - $\frac{13}{6}$, which is equivalent to $\frac{29}{6} + \frac{29}{6}$, or $\frac{29}{3}$. Note that 29/3, 9.666, and 9.667 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 511d1d81

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 511d1d81

$$(d-30)(d+30)-7=-7$$

What is a solution to the given equation?

ID: 511d1d81 Answer

Correct Answer: 30, -30

Rationale

The correct answer is either -30 or 30. Adding 7 to each side of the given equation yields d - $30d+30=0$. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either d - $30=0$ or $d+30=0$. Adding 30 to each side of the equation d - $30=0$ yields $d=30$. Subtracting 30 from each side of the equation $d+30=0$ yields $d=$ - 30. Therefore, the solutions to the given equation are -30 and 30. Note that -30 and 30 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID b4379e93

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: b4379e93

$$x^2 + y + 7 = 7$$
$$20x + 100 - y = 0$$

The solution to the given system of equations is (x, y). What is the value of x?

ID: b4379e93 Answer

Correct Answer: -10

Rationale

The correct answer is -10. Adding y to both sides of the second equation in the given system yields $20x + 100 = y$. Substituting $20x + 100$ for y in the first equation in the given system yields $x^2 + 20x + 100 + 7 = 7$. Subtracting 7 from both sides of this equation yields $x^2 + 20x + 100 = 0$. Factoring the left-hand side of this equation yields $x + 10x + 10 = 0$, or $x + 10^2 = 0$. Taking the square root of both sides of this equation yields $x + 10 = 0$. Subtracting 10 from both sides of this equation yields $x = -10$. Therefore, the value of x is -10.

Question Difficulty: Hard

Question ID 8294e05a

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 8294e05a

$$\sqrt{k-x} = 58 - x$$

In the given equation, k is a constant. The equation has exactly one real solution. What is the minimum possible value of $4k$?

ID: 8294e05a Answer

Correct Answer: 231

Rationale

The correct answer is 231. It's given that $\sqrt{k - x} = 58 - x$. Squaring both sides of this equation yields $k - x = (58 - x)^2$, which is equivalent to the given equation if $58 - x > 0$. It follows that if a solution to the equation $k - x = (58 - x)^2$ satisfies $58 - x > 0$, then it's also a solution to the given equation; if not, it's extraneous. The equation $k - x = (58 - x)^2$ can be rewritten as $k - x = 3{,}364 - 116x + x^2$. Adding x to both sides of this equation yields $k = x^2 - 115x + 3{,}364$. Subtracting k from both sides of this equation yields $0 = x^2 - 115x + (3{,}364 - k)$. The number of solutions to a quadratic equation in the form $0 = ax^2 + bx + c$, where a, b, and c are constants, can be determined by the value of the discriminant, $b^2 - 4ac$. Substituting -115 for b, 1 for a, and $3{,}364 - k$ for c in $b^2 - 4ac$ yields $(-115)^2 - 4(1)(3{,}364 - k)$, or $4k - 231$. The equation $0 = x^2 - 115x + (3{,}364 - k)$ has exactly one real solution if the discriminant is equal to zero, or $4k - 231 = 0$. Subtracting 231 from both sides of this equation yields $4k = 231$. Dividing both sides of this equation by 4 yields $k = 57.75$. Therefore, if $k = 57.75$, then the equation $0 = x^2 - 115x + (3{,}364 - k)$ has exactly one real solution. Substituting 57.75 for k in this equation yields $0 = x^2 - 115x + (3{,}364 - 57.75)$, or $0 = x^2 - 115x + 3{,}306.25$, which is equivalent to $0 = (x - 57.5)^2$. Taking the square root of both sides of this equation yields $0 = x - 57.5$. Adding 57.5 to both sides of this equation yields $57.5 = x$. To check whether this solution satisfies $58 - x > 0$, the solution, 57.5, can be substituted for x in $58 - x > 0$, which yields $58 - 57.5 > 0$, or $0.5 > 0$. Since 0.5 is greater than 0, it follows that if $k = 57.75$, or $4k = 231$, then the given equation has exactly one real solution. If $4k < 231$, then the discriminant, $4k - 231$, is negative and the given equation has no solutions. Therefore, the minimum possible value of $4k$ is 231.

Question Difficulty: Hard

Question ID a0296e89

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a0296e89

Which quadratic equation has no real solutions?

A. $x^2 + 14x - 49 = 0$

B. $x^2 - 14x + 49 = 0$

C. $5x^2 - 14x - 49 = 0$

D. $5x^2 - 14x + 49 = 0$

ID: a0296e89 Answer

Correct Answer: D

Rationale

Choice D is correct. The number of solutions to a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be determined by the value of the discriminant, $b^2 - 4ac$. If the value of the discriminant is greater than zero, then the quadratic equation has two distinct real solutions. If the value of the discriminant is equal to zero, then the quadratic equation has exactly one real solution. If the value of the discriminant is less than zero, then the quadratic equation has no real solutions. For the quadratic equation in choice D, $5x^2 - 14x + 49 = 0$, $a = 5$, $b = -14$, and $c = 49$. Substituting 5 for a, -14 for b, and 49 for c in $b^2 - 4ac$ yields $(-14)^2 - 4(5)(49)$, or -784. Since -784 is less than zero, it follows that the quadratic equation $5x^2 - 14x + 49 = 0$ has no real solutions.

Choice A is incorrect. The value of the discriminant for this quadratic equation is 392. Since 392 is greater than zero, it follows that this quadratic equation has two real solutions.

Choice B is incorrect. The value of the discriminant for this quadratic equation is 0. Since zero is equal to zero, it follows that this quadratic equation has exactly one real solution.

Choice C is incorrect. The value of the discriminant for this quadratic equation is $1{,}176$. Since $1{,}176$ is greater than zero, it follows that this quadratic equation has two real solutions.

Question Difficulty: Hard

Question ID a740186b

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: a740186b

$$\frac{20}{p} = \frac{20}{q} - \frac{20}{r} - \frac{20}{s}$$

The given equation relates the positive variables p, q, r, and s. Which of the following is equivalent to q?

A. $p + r + s$

B. $20(p + r + s)$

C. $\frac{prs}{pr+ps+rs}$

D. $\frac{prs}{20p+20r+20s}$

ID: a740186b Answer

Correct Answer: C

Rationale

Choice C is correct. Multiplying each side of the given equation by $\frac{1}{20}$ yields $\frac{1}{20}\frac{20}{p} = \frac{1}{20}\frac{20}{q} - \frac{20}{r} - \frac{20}{s}$. Distributing $\frac{1}{20}$ on each side of this equation yields $\frac{20}{20p} = \frac{20}{20q} - \frac{20}{20r} - \frac{20}{20s}$, or $\frac{1}{p} = \frac{1}{q} - \frac{1}{r} - \frac{1}{s}$. Adding $\frac{1}{r} + \frac{1}{s}$ to each side of this equation yields $\frac{1}{s} + \frac{1}{r} + \frac{1}{p} = \frac{1}{q}$. Multiplying $\frac{1}{s}$ by $\frac{pr}{pr}$, $\frac{1}{r}$ by $\frac{ps}{ps}$, and $\frac{1}{p}$ by $\frac{rs}{rs}$ yields $\frac{pr}{prs} + \frac{ps}{prs} + \frac{rs}{prs} = \frac{1}{q}$, which is equivalent to $\frac{pr+ps+rs}{prs} = \frac{1}{q}$. Since $\frac{pr+ps+rs}{prs} = \frac{1}{q}$, and it's given that p, q, r, and s are positive, it follows that the reciprocals of each side of this equation are also equal. Thus, $\frac{prs}{pr+ps+rs} = \frac{q}{1}$, or $\frac{prs}{pr+ps+rs} = q$. Therefore, $\frac{prs}{pr+ps+rs}$ is equivalent to q.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2d5ffbc6

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 2d5ffbc6

$$y = -1.5$$
$$y = x^2 + 8x + a$$

In the given system of equations, a is a positive constant. The system has exactly one distinct real solution. What is the value of a ?

ID: 2d5ffbc6 Answer

Correct Answer: 14.5, 29/2

Rationale

The correct answer is $\frac{29}{2}$. According to the first equation in the given system, the value of y is -1.5. Substituting -1.5 for y in the second equation in the given system yields $-1.5 = x^2 + 8x + a$. Adding 1.5 to both sides of this equation yields $0 = x^2 + 8x + a + 1.5$. If the given system has exactly one distinct real solution, it follows that $0 = x^2 + 8x + a + 1.5$ has exactly one distinct real solution. A quadratic equation in the form $0 = px^2 + qx + r$, where p, q, and r are constants, has exactly one distinct real solution if and only if the discriminant, $q^2 - 4pr$, is equal to 0. The equation $0 = x^2 + 8x + a + 1.5$ is in this form, where $p = 1$, $q = 8$, and $r = a + 1.5$. Therefore, the discriminant of the equation $0 = x^2 + 8x + a + 1.5$ is $8^2 - 41a + 1.5$, or $58 - 4a$. Setting the discriminant equal to 0 to solve for a yields $58 - 4a = 0$. Adding $4a$ to both sides of this equation yields $58 = 4a$. Dividing both sides of this equation by 4 yields $\frac{58}{4} = a$, or $\frac{29}{2} = a$. Therefore, if the given system of equations has exactly one distinct real solution, the value of a is $\frac{29}{2}$. Note that 29/2 and 14.5 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID 6b439b01

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 6b439b01

$$x^2 + y + 10 = 10$$
$$8x + 16 - y = 0$$

The solution to the given system of equations is (x, y). What is the value of x?

A. -16

B. -4

C. 2

D. 8

ID: 6b439b01 Answer

Correct Answer: B

Rationale

Choice B is correct. Adding y to each side of the second equation in the given system of equations yields $8x + 16 = y$. Substituting $8x + 16$ for y in the first equation yields $x^2 + 8x + 16 + 10 = 10$. Subtracting 10 from each side of this equation yields $x^2 + 8x + 16 = 0$. This equation can be rewritten as $x + 4^2 = 0$. Taking the square root of each side of this equation yields $x + 4 = 0$. Subtracting 4 from each side of this equation yields $x =$ -4. Therefore, the value of x is -4.

Choice A is incorrect. This is the value of y, not x.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 0dd61384

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 0dd61384

If $3x^2 - 18x - 15 = 0$, what is the value of $x^2 - 6x$?

ID: 0dd61384 Answer

Correct Answer: 5

Rationale

The correct answer is 5. Dividing each side of the given equation by 3 yields $x^2 - 6x - 5 = 0$. Adding 5 to each side of this equation yields $x^2 - 6x = 5$. Therefore, if $3x^2 - 18x - 15 = 0$, the value of $x^2 - 6x$ is 5.

Question Difficulty: Hard

Question ID 3aaea513

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 3aaea513

$$-4x^2 - 7x = -36$$

What is the positive solution to the given equation?

A. $\frac{7}{4}$

B. $\frac{9}{4}$

C. 4

D. 7

ID: 3aaea513 Answer

Correct Answer: B

Rationale

Choice B is correct. Multiplying each side of the given equation by -16 yields $64x^2 + 112x = 576$. To complete the square, adding 49 to each side of this equation yields $64x^2 + 112x + 49 = 576 + 49$, or $8x + 7^2 = 625$. Taking the square root of each side of this equation yields two equations: $8x + 7 = 25$ and $8x + 7 =$ - 25. Subtracting 7 from each side of the equation $8x + 7 = 25$ yields $8x = 18$. Dividing each side of this equation by 8 yields $x = \frac{18}{8}$, or $x = \frac{9}{4}$. Therefore, $\frac{9}{4}$ is a solution to the given equation. Subtracting 7 from each side of the equation $8x + 7 =$ - 25 yields $8x =$ - 32. Dividing each side of this equation by 8 yields $x =$ - 4 . Therefore, the given equation has two solutions, $\frac{9}{4}$ and -4. Since $\frac{9}{4}$ is positive, it follows that $\frac{9}{4}$ is the positive solution to the given equation.

Alternate approach: Adding $4x^2$ and $7x$ to each side of the given equation yields $0 = 4x^2 + 7x$ - 36. The right-hand side of this equation can be rewritten as $4x^2 + 16x$ - $9x$ - 36. Factoring out the common factor of $4x$ from the first two terms of this expression and the common factor of -9 from the second two terms yields $4xx + 4$ - $9x + 4$. Factoring out the common factor of $x + 4$ from these two terms yields the expression $4x$ - $9x + 4$. Since this expression is equal to 0, it follows that either $4x$ - $9 = 0$ or $x + 4 = 0$. Adding 9 to each side of the equation $4x$ - $9 = 0$ yields $4x = 9$. Dividing each side of this equation by 4 yields $x = \frac{9}{4}$. Therefore, $\frac{9}{4}$ is a positive solution to the given equation. Subtracting 4 from each side of the equation $x + 4 = 0$ yields $x =$ - 4. Therefore, the given equation has two solutions, $\frac{9}{4}$ and -4. Since $\frac{9}{4}$ is positive, it follows that $\frac{9}{4}$ is the positive solution to the given equation.

Choice A is incorrect. Substituting $\frac{7}{4}$ for x in the given equation yields -$\frac{49}{2} =$ - 36, which is false.

Choice C is incorrect. Substituting 4 for x in the given equation yields -92 $=$ - 36, which is false.

Choice D is incorrect. Substituting 7 for x in the given equation yields -245 $=$ - 36, which is false.

Question Difficulty: Hard

Question ID 1fbc1bcd

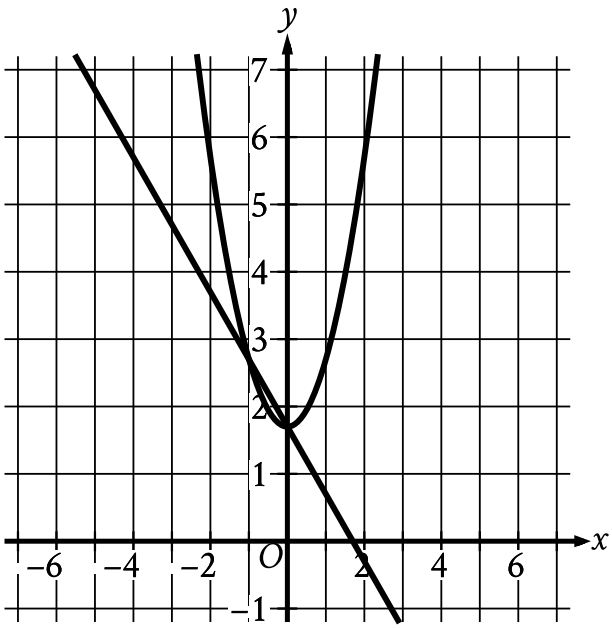
Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 1fbc1bcd

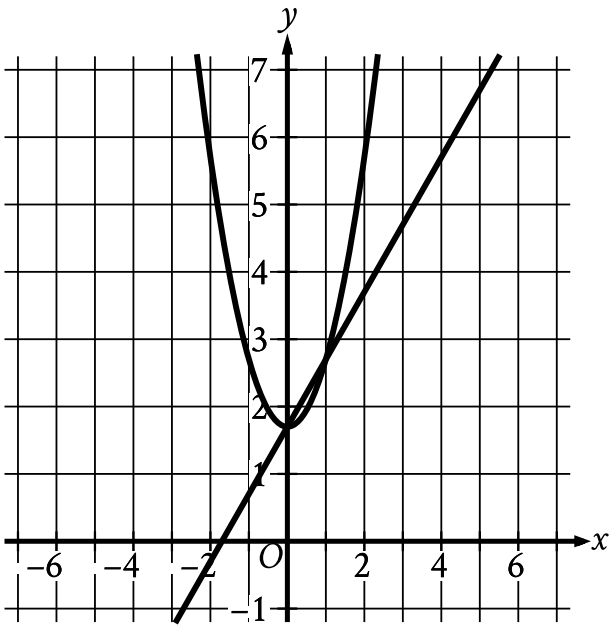
$$y = x^2 + 1.7$$
$$y = 1.7 - x$$

Which graph represents the given system of equations?

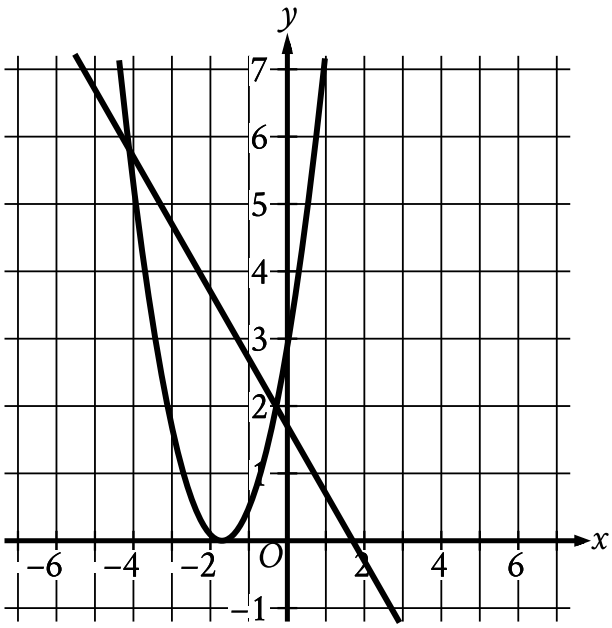
A.



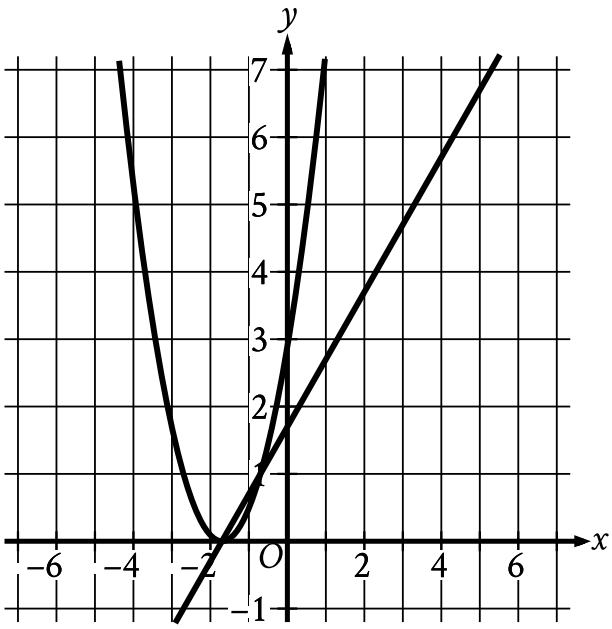
B.



C.



D.



ID: 1fbc1bcd Answer

Correct Answer: A

Rationale

Choice A is correct. The graph of a quadratic equation in the form $y = x^2 + c$ has its vertex at $(0, c)$. The first equation in the given system of equations is $y = x^2 + 1.7$, so the graph of this quadratic equation has its vertex at $(0, 1.7)$. The graph of a linear equation of the form $y = b - x$ has a slope of -1 and a *y*-intercept at $(0, b)$. The second equation in the given system of equations is $y = 1.7 - x$, so the graph of this linear equation has a slope of -1 and a *y*-intercept at $(0, 1.7)$. Of the choices, only choice A has the graph of a quadratic equation with its vertex at $(0, 1.7)$ and the graph of a linear equation with a slope of -1 and a *y*-intercept at $(0, 1.7)$.

Choice B is incorrect. This graph represents a system in which the second equation is $y = 1.7 + x$, not $y = 1.7 - x$.

Choice C is incorrect. This graph represents a system in which the first equation is $y = (x + 1.7)^2$, not $y = x^2 + 1.7$.

Choice D is incorrect. This graph represents a system in which the first equation is $y = (x + 1.7)^2$, not $y = x^2 + 1.7$, and the second equation is $y = 1.7 + x$, not $y = 1.7 - x$.

Question Difficulty: Hard

Question ID cfe8bac6

Assessment	Test	Domain	Skill	Difficulty
PSAT 8/9	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: cfe8bac6

$$57x^2 + (57b + a)x + ab = 0$$

In the given equation, a and b are positive constants. The product of the solutions to the given equation is kab, where k is a constant. What is the value of k?

A. $\frac{1}{57}$

B. $\frac{1}{19}$

C. 1

D. 57

ID: cfe8bac6 Answer

Correct Answer: A

Rationale

Choice A is correct. The left-hand side of the given equation is the expression $57x^2 + 57b + ax + ab$. Applying the distributive property to this expression yields $57x^2 + 57bx + ax + ab$. Since the first two terms of this expression have a common factor of $57x$ and the last two terms of this expression have a common factor of a, this expression can be rewritten as $57xx + b + ax + b$. Since the two terms of this expression have a common factor of $x + b$, it can be rewritten as $x + b57x + a$. Therefore, the given equation can be rewritten as $x + b57x + a = 0$. By the zero product property, it follows that $x + b = 0$ or $57x + a = 0$. Subtracting b from both sides of the equation $x + b = 0$ yields $x = \text{-}\ b$. Subtracting a from both sides of the equation $57x + a = 0$ yields $57x = \text{-}\ a$. Dividing both sides of this equation by 57 yields $x = \frac{\text{-}a}{57}$. Therefore, the solutions to the given equation are -b and $\frac{\text{-}a}{57}$. It follows that the product of the solutions of the given equation is -$b\frac{\text{-}a}{57}$, or $\frac{ab}{57}$. It's given that the product of the solutions of the given equation is kab. It follows that $\frac{ab}{57} = kab$, which can also be written as $ab\frac{1}{57} = abk$. It's given that a and b are positive constants. Therefore, dividing both sides of the equation $ab\frac{1}{57} = abk$ by ab yields $\frac{1}{57} = k$. Thus, the value of k is $\frac{1}{57}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard