

Question ID e8a6c1fc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: e8a6c1fc

What is the value of $\tan \frac{92\pi}{3}$?

- A. $-\sqrt{3}$
- B. $-\frac{\sqrt{3}}{3}$
- C. $\frac{\sqrt{3}}{3}$
- D. $\sqrt{3}$

ID: e8a6c1fc Answer

Correct Answer: A

Rationale

Choice A is correct. A trigonometric ratio can be found using the unit circle, that is, a circle with radius 1 unit. If a central angle of a unit circle in the xy -plane centered at the origin has its starting side on the positive x -axis and its terminal side intersects the circle at a point x, y , then the value of the tangent of the central angle is equal to the y -coordinate divided by the x -coordinate. There are 2π radians in a circle. Dividing $\frac{92\pi}{3}$ by 2π yields $\frac{92}{6}$, which is equivalent to $15 + \frac{2}{3}$. It follows that on the unit circle centered at the origin in the xy -plane, the angle $\frac{92\pi}{3}$ is the result of 15 revolutions from its starting side on the positive x -axis followed by a rotation through $\frac{2\pi}{3}$ radians. Therefore, the angles $\frac{92\pi}{3}$ and $\frac{2\pi}{3}$ are coterminal angles and $\tan \frac{92\pi}{3}$ is equal to $\tan \frac{2\pi}{3}$. Since $\frac{2\pi}{3}$ is greater than $\frac{\pi}{2}$ and less than π , it follows that the terminal side of the angle is in quadrant II and forms an angle of $\frac{\pi}{3}$, or 60° , with the negative x -axis. Therefore, the terminal side of the angle intersects the unit circle at the point $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$. It follows that the value of $\tan \frac{2\pi}{3}$ is $\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$, which is equivalent to $-\sqrt{3}$. Therefore, the value of $\tan \frac{92\pi}{3}$ is $-\sqrt{3}$.

Choice B is incorrect. This is the value of $\frac{1}{\tan \frac{92\pi}{3}}$, not $\tan \frac{92\pi}{3}$.

Choice C is incorrect. This is the value of $\frac{1}{\tan \frac{\pi}{3}}$, not $\tan \frac{92\pi}{3}$.

Choice D is incorrect. This is the value of $\tan \frac{\pi}{3}$, not $\tan \frac{92\pi}{3}$.

Question Difficulty: Hard

Question ID 9f2728be

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9f2728be

The graph of $x^2 + x + y^2 + y = \frac{199}{2}$ in the xy -plane is a circle. What is the length of the circle's radius?

ID: 9f2728be Answer

Correct Answer: 10

Rationale

The correct answer is 10. It's given that the graph of $x^2 + x + y^2 + y = \frac{199}{2}$ in the xy -plane is a circle. The equation of a circle in the xy -plane can be written in the form $x - h^2 + y - k^2 = r^2$, where the coordinates of the center of the circle are h, k and the length of the radius of the circle is r . The term $x - h^2$ in this equation can be obtained by adding the square of half the coefficient of x to both sides of the given equation to complete the square. The coefficient of x is 1. Half the coefficient of x is $\frac{1}{2}$. The square of half the coefficient of x is $\frac{1}{4}$. Adding $\frac{1}{4}$ to each side of $x^2 + x + y^2 + y = \frac{199}{2}$ yields $x^2 + x + \frac{1}{4} + y^2 + y = \frac{199}{2} + \frac{1}{4}$, or $x + \frac{1^2}{2} + y^2 + y = \frac{199}{2} + \frac{1}{4}$. Similarly, the term $y - k^2$ can be obtained by adding the square of half the coefficient of y to both sides of this equation, which yields $x + \frac{1^2}{2} + y^2 + y + \frac{1}{4} = \frac{199}{2} + \frac{1}{4} + \frac{1}{4}$, or $x + \frac{1^2}{2} + y + \frac{1^2}{2} = \frac{199}{2} + \frac{1}{4} + \frac{1}{4}$. This equation is equivalent to $x + \frac{1^2}{2} + y + \frac{1^2}{2} = 100$, or $x + \frac{1^2}{2} + y + \frac{1^2}{2} = 10^2$. Therefore, the length of the circle's radius is 10.

Question Difficulty: Hard

Question ID fde10025

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fde10025

A circle in the xy -plane has its center at $(-1, 1)$. Line t is tangent to this circle at the point $(5, -4)$. Which of the following points also lies on line t ?

- A. $(0, \frac{6}{5})$
- B. $(4, 7)$
- C. $(10, 2)$
- D. $(11, 1)$

ID: fde10025 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the circle has its center at $-1, 1$ and that line t is tangent to this circle at the point $5, -4$. Therefore, the points $-1, 1$ and $5, -4$ are the endpoints of the radius of the circle at the point of tangency. The slope of a line or line segment that contains the points a, b and c, d can be calculated as $\frac{d-b}{c-a}$. Substituting $-1, 1$ for a, b and $5, -4$ for c, d in the expression $\frac{d-b}{c-a}$ yields $\frac{-4-1}{5-(-1)}$, or $-\frac{5}{6}$. Thus, the slope of this radius is $-\frac{5}{6}$. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It follows that line t is perpendicular to the radius at the point $5, -4$, so the slope of line t is the negative reciprocal of the slope of this radius. The negative reciprocal of $-\frac{5}{6}$ is $\frac{6}{5}$. Therefore, the slope of line t is $\frac{6}{5}$. Since the slope of line t is the same between any two points on line t , a point lies on line t if the slope of the line segment connecting the point and $5, -4$ is $\frac{6}{5}$. Substituting choice C, $10, 2$, for a, b and $5, -4$ for c, d in the expression $\frac{d-b}{c-a}$ yields $\frac{-4-2}{5-10}$, or $\frac{6}{5}$. Therefore, the point $10, 2$ lies on line t .

Choice A is incorrect. The slope of the line segment connecting $0, \frac{6}{5}$ and $5, -4$ is $\frac{-4-\frac{6}{5}}{5-0}$, or $-\frac{26}{25}$, not $\frac{6}{5}$.

Choice B is incorrect. The slope of the line segment connecting $4, 7$ and $5, -4$ is $\frac{-4-7}{5-4}$, or -11 , not $\frac{6}{5}$.

Choice D is incorrect. The slope of the line segment connecting $11, 1$ and $5, -4$ is $\frac{-4-1}{5-11}$, or $\frac{5}{6}$, not $\frac{6}{5}$.

Question Difficulty: Hard

Question ID 32f6a450

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 32f6a450

What is the diameter of the circle in the xy -plane with equation $(x - 5)^2 + (y - 3)^2 = 16$?

- A. 4
- B. 8
- C. 16
- D. 32

ID: 32f6a450 Answer

Correct Answer: B

Rationale

Choice B is correct. The standard form of an equation of a circle in the xy -plane is $x - h^2 + y - k^2 = r^2$, where the coordinates of the center of the circle are h, k and the length of the radius of the circle is r . For the circle in the xy -plane with equation $x - 5^2 + y - 3^2 = 16$, it follows that $r^2 = 16$. Taking the square root of both sides of this equation yields $r = 4$ or $r = - 4$. Because r represents the length of the radius of the circle and this length must be positive, $r = 4$. Therefore, the radius of the circle is 4. The diameter of a circle is twice the length of the radius of the circle. Thus, 24 yields 8. Therefore, the diameter of the circle is 8.

Choice A is incorrect. This is the radius of the circle.

Choice C is incorrect. This is the square of the radius of the circle.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b006acd3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: b006acd3

Circle A has equation $(x - 7)^2 + (y + 3)^2 = 1$. In the xy -plane, circle B is obtained by translating circle A to the right 4 units. Which equation represents circle B?

- A. $(x - 7)^2 + (y + 7)^2 = 1$
- B. $(x - 3)^2 + (y + 3)^2 = 1$
- C. $(x - 11)^2 + (y + 3)^2 = 1$
- D. $(x - 7)^2 + (y - 1)^2 = 1$

ID: b006acd3 Answer

Correct Answer: C

Rationale

Choice C is correct. The equation of a circle in the xy -plane can be written as $(x - h)^2 + (y - k)^2 = r^2$, where the center of the circle is (h, k) and the radius of the circle is r units. It's given that circle A has the equation $(x - 7)^2 + (y + 3)^2 = 1$, which can be written as $(x - 7)^2 + (y - (-3))^2 = 1^2$. It follows that $h = 7$, $k = -3$, and $r = 1$. Therefore, the center of circle A is $(7, -3)$ and its radius is 1 unit. If circle A is translated 4 units to the right, the x -coordinate of the center will increase by 4, while the y -coordinate and the radius of the circle will remain unchanged. Translating the center of circle A to the right 4 units yields $(7 + 4, -3)$, or $(11, -3)$. Therefore, the center of circle B is $(11, -3)$. Substituting 11 for h , -3 for k , and 1 for r into the equation $(x - h)^2 + (y - k)^2 = r^2$ yields $(x - 11)^2 + (y - (-3))^2 = 1$, or $(x - 11)^2 + (y + 3)^2 = 1$. Therefore, the equation $(x - 11)^2 + (y + 3)^2 = 1$ represents circle B.

Choice A is incorrect. This equation represents a circle obtained by shifting circle A down, rather than right, 4 units.

Choice B is incorrect. This equation represents a circle obtained by shifting circle A left, rather than right, 4 units.

Choice D is incorrect. This equation represents a circle obtained by shifting circle A up, rather than right, 4 units.

Question Difficulty: Hard

Question ID 82372955

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 82372955

In the xy -plane, a circle has center C with coordinates (h, k) . Points A and B lie on the circle. Point A has coordinates $(h + 1, k + \sqrt{102})$, and $\angle ACB$ is a right angle. What is the length of $\overline{AB}$?

- A. $\sqrt{206}$
- B. $2\sqrt{102}$
- C. $103\sqrt{2}$
- D. $103\sqrt{3}$

ID: 82372955 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that points A and B lie on the circle with center C . Therefore, $\overline{AC}$ and $\overline{BC}$ are both radii of the circle. Since all radii of a circle are congruent, $\overline{AC}$ is congruent to $\overline{BC}$. The length of $\overline{AC}$, or the distance from point A to point C , can be found using the distance formula, which gives the distance between two points, x_1, y_1 and x_2, y_2 , as $\sqrt{x_1 - x_2^2 + y_1 - y_2^2}$. Substituting the given coordinates of point A , $h + 1, k + \sqrt{102}$, for x_1, y_1 and the given coordinates of point C , h, k , for x_2, y_2 in the distance formula yields $\sqrt{h + 1 - h^2 + k + \sqrt{102} - k^2}$, or $\sqrt{1^2 + \sqrt{102}^2}$, which is equivalent to $\sqrt{1 + 102}$, or $\sqrt{103}$. Therefore, the length of $\overline{AC}$ is $\sqrt{103}$ and the length of $\overline{BC}$ is $\sqrt{103}$. It's given that angle ACB is a right angle. Therefore, triangle ACB is a right triangle with legs $\overline{AC}$ and $\overline{BC}$ and hypotenuse $\overline{AB}$. By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b , then $a^2 + b^2 = c^2$. Substituting $\sqrt{103}$ for a and b in this equation yields $\sqrt{103}^2 + \sqrt{103}^2 = c^2$, or $103 + 103 = c^2$, which is equivalent to $206 = c^2$. Taking the positive square root of both sides of this equation yields $\sqrt{206} = c$. Therefore, the length of $\overline{AB}$ is $\sqrt{206}$.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This would be the length of $\overline{AB}$ if the length of $\overline{AC}$ were 103, not $\sqrt{103}$.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 40789a56

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 40789a56

Circle A in the xy -plane has the equation $(x + 5)^2 + (y - 5)^2 = 4$. Circle B has the same center as circle A. The radius of circle B is two times the radius of circle A. The equation defining circle B in the xy -plane is $(x + 5)^2 + (y - 5)^2 = k$, where k is a constant. What is the value of k ?

ID: 40789a56 Answer

Correct Answer: 16

Rationale

The correct answer is 16. An equation of a circle in the xy -plane can be written as $x - t^2 + y - u^2 = r^2$, where the center of the circle is t, u , the radius of the circle is r , and where t, u , and r are constants. It's given that the equation of circle A is $x + 5^2 + y - 5^2 = 4$, which is equivalent to $x + 5^2 + y - 5^2 = 2^2$. Therefore, the center of circle A is $-5, 5$ and the radius of circle A is 2. It's given that circle B has the same center as circle A and that the radius of circle B is two times the radius of circle A. Therefore, the center of circle B is $-5, 5$ and the radius of circle B is 22, or 4. Substituting -5 for t , 5 for u , and 4 for r into the equation $x - t^2 + y - u^2 = r^2$ yields $x + 5^2 + y - 5^2 = 4^2$, which is equivalent to $x + 5^2 + y - 5^2 = 16$. It follows that the equation of circle B in the xy -plane is $x + 5^2 + y - 5^2 = 16$. Therefore, the value of k is 16.

Question Difficulty: Hard

Question ID 409da8a2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 409da8a2

A circle in the xy -plane has its center at $(16, 17)$ and has a radius of $7k$. Which equation represents this circle?

- A. $(x - 16)^2 + (y - 17)^2 = 49k$
- B. $(x - 16)^2 + (y - 17)^2 = 49k^2$
- C. $(x - 16)^2 + (y - 17)^2 = 7k$
- D. $(x - 16)^2 + (y - 17)^2 = 7k^2$

ID: 409da8a2 Answer

Correct Answer: B

Rationale

Choice B is correct. The equation of a circle in the xy -plane can be written as $(x - h)^2 + (y - k)^2 = r^2$, where the center of the circle is (h, k) and the radius of the circle is r . It's given that this circle has a center at $(16, 17)$ and a radius of $7k$. Substituting 16 for h , 17 for k , and $7k$ for r in $(x - h)^2 + (y - k)^2 = r^2$ yields $(x - 16)^2 + (y - 17)^2 = (7k)^2$, or $(x - 16)^2 + (y - 17)^2 = 49k^2$. Therefore, the equation that represents this circle is $(x - 16)^2 + (y - 17)^2 = 49k^2$.

Choice A is incorrect. This equation represents a circle with radius $7\sqrt{k}$, not $7k$.

Choice C is incorrect. This equation represents a circle with radius $\sqrt{7k}$, not $7k$.

Choice D is incorrect. This equation represents a circle with radius $\sqrt{7}k$, not $7k$.

Question Difficulty: Hard

Question ID 33be0f76

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 33be0f76

The equation $x^2 + (y - 2)^2 = 36$ represents circle A. Circle B is obtained by shifting circle A down 4 units in the xy -plane. Which of the following equations represents circle B?

- A. $x^2 + (y + 2)^2 = 36$
- B. $x^2 + (y - 6)^2 = 36$
- C. $(x - 4)^2 + (y - 2)^2 = 36$
- D. $(x + 4)^2 + (y - 2)^2 = 36$

ID: 33be0f76 Answer

Correct Answer: A

Rationale

Choice A is correct. The standard form of an equation of a circle in the xy -plane is $x - h^2 + y - k^2 = r^2$, where the coordinates of the center of the circle are h, k and the length of the radius of the circle is r . The equation of circle A, $x^2 + y - 2^2 = 36$, can be rewritten as $x - 0^2 + y - 2^2 = 6^2$. Therefore, the center of circle A is at 0, 2 and the length of the radius of circle A is 6. If circle A is shifted down 4 units, the y -coordinate of its center will decrease by 4; the radius of the circle and the x -coordinate of its center will not change. Therefore, the center of circle B is at 0, 2 - 4, or 0, - 2, and its radius is 6. Substituting 0 for h , -2 for k , and 6 for r in the equation $x - h^2 + y - k^2 = r^2$ yields $x - 0^2 + y - -2^2 = 6^2$, or $x^2 + y + 2^2 = 36$. Therefore, the equation $x^2 + y + 2^2 = 36$ represents circle B.

Choice B is incorrect. This equation represents a circle obtained by shifting circle A up, rather than down, 4 units.

Choice C is incorrect. This equation represents a circle obtained by shifting circle A right, rather than down, 4 units.

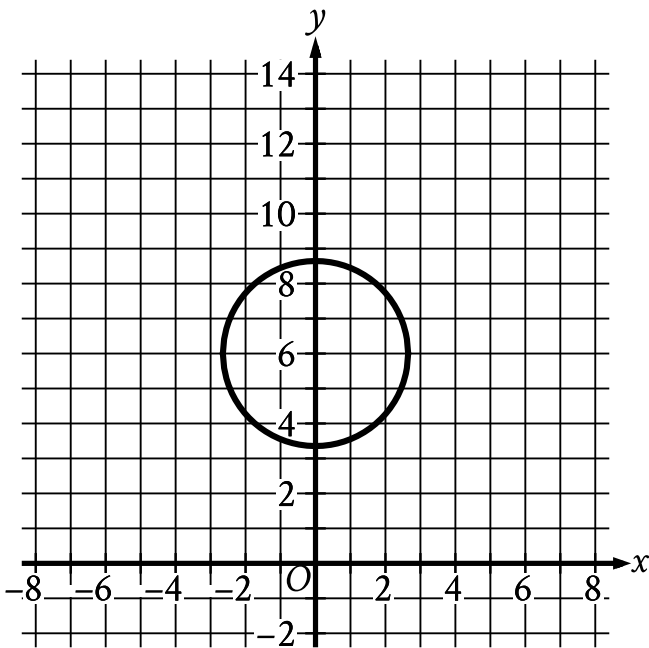
Choice D is incorrect. This equation represents a circle obtained by shifting circle A left, rather than down, 4 units.

Question Difficulty: Hard

Question ID 13df6820

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 13df6820



Circle A shown is defined by the equation $x^2 + (y - 6)^2 = 7$. Circle B (not shown) has the same radius but is translated 96 units to the right. If the equation of circle B is $(x - h)^2 + (y - k)^2 = a$, where h , k , and a are constants, what is the value of $4a$?

ID: 13df6820 Answer

Correct Answer: 28

Rationale

The correct answer is 28. The equation of a circle in the xy -plane can be written as $(x - t)^2 + (y - s)^2 = r^2$, where the center of the circle is (t, s) and the radius of the circle is r . It's given that circle A is defined by the equation $x^2 + (y - 6)^2 = 7$, which can be written as $(x - 0)^2 + (y - 6)^2 = \sqrt{7}^2$. It follows that $r = \sqrt{7}$ and the radius of circle A is $\sqrt{7}$. It's also given that circle B has the same radius as circle A. If the equation of circle B is $(x - h)^2 + (y - k)^2 = a$, then $a = r^2$. Substituting $\sqrt{7}$ for r in this equation yields $a = \sqrt{7}^2$, or $a = 7$. It follows that the value of $4a$ is $4(7)$, or 28.

Question Difficulty: Hard

Question ID fd77ca64

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fd77ca64

$$x^2 + 14x + y^2 = 6y + 109$$

In the xy -plane, the graph of the given equation is a circle. What is the length of the circle's radius?

- A. $\sqrt{109}$
- B. $\sqrt{149}$
- C. $\sqrt{167}$
- D. $\sqrt{341}$

ID: fd77ca64 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that in the xy -plane, the graph of the given equation is a circle. The equation of a circle in the xy -plane can be written in the form $x - h^2 + y - k^2 = r^2$, where h, k is the center of the circle and r is the length of the circle's radius. Subtracting $6y$ from both sides of the equation $x^2 + 14x + y^2 = 6y + 109$ yields $x^2 + 14x + y^2 - 6y = 109$. By completing the square, this equation can be rewritten as $x^2 + 14x + \frac{14^2}{2} + y^2 - 6y + \frac{-6^2}{2} = 109 + \frac{14^2}{2} + \frac{-6^2}{2}$. This equation can be rewritten as $x^2 + 14x + 49 + y^2 - 6y + 9 = 109 + 49 + 9$, or $x + 7^2 + y - 3^2 = 167$. Therefore, $r^2 = 167$. Taking the square root of both sides of this equation yields $r = \sqrt{167}$ and $r = -\sqrt{167}$. Since r is the length of the circle's radius, r must be positive. Therefore, the length of the circle's radius is $\sqrt{167}$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID ea7fd37b

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: ea7fd37b

$$(x + 4)^2 + (y - 19)^2 = 121$$

The graph of the given equation is a circle in the xy -plane. The point (a, b) lies on the circle. Which of the following is a possible value for a ?

- A. -16
- B. -14
- C. 11
- D. 19

ID: ea7fd37b Answer

Correct Answer: B

Rationale

Choice B is correct. An equation of the form $x - h^2 + y - k^2 = r^2$, where h, k , and r are constants, represents a circle in the xy -plane with center h, k and radius r . Therefore, the circle represented by the given equation has center $-4, 19$ and radius 11 . Since the center of the circle has an x -coordinate of -4 and the radius of the circle is 11 , the least possible x -coordinate for any point on the circle is $-4 - 11$, or -15 . Similarly, the greatest possible x -coordinate for any point on the circle is $-4 + 11$, or 7 . Therefore, if the point a, b lies on the circle, it must be true that $-15 \leq a \leq 7$. Of the given choices, only -14 satisfies this inequality.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID fb2d9203

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: fb2d9203

Points Q and R lie on a circle with center P . The radius of this circle is 9 inches. Triangle PQR has a perimeter of 31 inches. What is the length, in inches, of $\overline{QR}$?

- A. $13\sqrt{2}$
- B. 13
- C. $9\sqrt{2}$
- D. 9

ID: fb2d9203 Answer

Correct Answer: B

Rationale

Choice B is correct. Since it's given that P is the center of a circle with a radius of 9 inches, and that points Q and R lie on that circle, it follows that $\overline{PQ}$ and $\overline{PR}$ of triangle PQR each have a length of 9 inches. Let the length of $\overline{QR}$ be x inches. It follows that the perimeter of triangle PQR is $9 + 9 + x$ inches. Since it's given that the perimeter of triangle PQR is 31 inches, it follows that $9 + 9 + x = 31$, or $18 + x = 31$. Subtracting 18 from both sides of this equation gives $x = 13$. Therefore, the length, in inches, of $\overline{QR}$ is 13.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b2aa5d73

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: b2aa5d73

A circle in the xy -plane has its center at $(-5, 2)$ and has a radius of 9. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a , b , and c are constants. What is the value of c ?

ID: b2aa5d73 Answer

Correct Answer: -52

Rationale

The correct answer is -52. The equation of a circle in the xy -plane with its center at h, k and a radius of r can be written in the form $x - h^2 + y - k^2 = r^2$. It's given that a circle in the xy -plane has its center at -5, 2 and has a radius of 9. Substituting -5 for h , 2 for k , and 9 for r in the equation $x - h^2 + y - k^2 = r^2$ yields $x - 5^2 + y - 2^2 = 9^2$, or $x + 5^2 + y - 2^2 = 81$. It's also given that an equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a , b , and c are constants. Therefore, $x + 5^2 + y - 2^2 = 81$ can be rewritten in the form $x^2 + y^2 + ax + by + c = 0$. The equation $x + 5^2 + y - 2^2 = 81$, or $x + 5x + 5 + y - 2y - 2 = 81$, can be rewritten as $x^2 + 5x + 5x + 25 + y^2 - 2y - 2y + 4 = 81$. Combining like terms on the left-hand side of this equation yields $x^2 + y^2 + 10x - 4y + 29 = 81$. Subtracting 81 from both sides of this equation yields $x^2 + y^2 + 10x - 4y - 52 = 0$, which is equivalent to $x^2 + y^2 + 10x + -4y + -52 = 0$. This equation is in the form $x^2 + y^2 + ax + by + c = 0$. Therefore, the value of c is -52.

Question Difficulty: Hard

Question ID 34eb7da8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 34eb7da8

The equation $x^2 + (y - 1)^2 = 49$ represents circle A. Circle B is obtained by shifting circle A down **2** units in the xy -plane. Which of the following equations represents circle B?

- A. $(x - 2)^2 + (y - 1)^2 = 49$
- B. $x^2 + (y - 3)^2 = 49$
- C. $(x + 2)^2 + (y - 1)^2 = 49$
- D. $x^2 + (y + 1)^2 = 49$

ID: 34eb7da8 Answer

Correct Answer: D

Rationale

Choice D is correct. The graph in the xy -plane of an equation of the form $x - h^2 + y - k^2 = r^2$ is a circle with center h, k and a radius of length r . It's given that circle A is represented by $x^2 + y - 1^2 = 49$, which can be rewritten as $x^2 + y - 1^2 = 7^2$. Therefore, circle A has center 0, 1 and a radius of length 7. Shifting circle A down two units is a rigid vertical translation of circle A that does not change its size or shape. Since circle B is obtained by shifting circle A down two units, it follows that circle B has the same radius as circle A, and for each point x, y on circle A, the point $x, y - 2$ lies on circle B. Moreover, if h, k is the center of circle A, then $h, k - 2$ is the center of circle B. Therefore, circle B has a radius of 7 and the center of circle B is 0, 1 - 2, or 0, - 1. Thus, circle B can be represented by the equation $x^2 + y + 1^2 = 7^2$, or $x^2 + y + 1^2 = 49$.

Choice A is incorrect. This is the equation of a circle obtained by shifting circle A right 2 units.

Choice B is incorrect. This is the equation of a circle obtained by shifting circle A up 2 units.

Choice C is incorrect. This is the equation of a circle obtained by shifting circle A left 2 units.

Question Difficulty: Hard

Question ID 8d56e2be

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 8d56e2be

A circle has center O , and points R and S lie on the circle. In triangle ORS , the measure of $\angle ROS$ is 88° . What is the measure of $\angle RSO$, in degrees? (Disregard the degree symbol when entering your answer.)

ID: 8d56e2be Answer

Correct Answer: 46

Rationale

The correct answer is 46. It's given that O is the center of a circle and that points R and S lie on the circle. Therefore, $\overline{OR}$ and $\overline{OS}$ are radii of the circle. It follows that $OR = OS$. If two sides of a triangle are congruent, then the angles opposite them are congruent. It follows that the angles $\angle RSO$ and $\angle ORS$, which are across from the sides of equal length, are congruent. Let x° represent the measure of $\angle RSO$. It follows that the measure of $\angle ORS$ is also x° . It's given that the measure of $\angle ROS$ is 88° . Because the sum of the measures of the interior angles of a triangle is 180° , the equation $x^\circ + x^\circ + 88^\circ = 180^\circ$, or $2x + 88 = 180$, can be used to find the measure of $\angle RSO$. Subtracting 88 from both sides of this equation yields $2x = 92$. Dividing both sides of this equation by 2 yields $x = 46$. Therefore, the measure of $\angle RSO$, in degrees, is 46.

Question Difficulty: Hard

Question ID ff2ced2c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: ff2ced2c

Which of the following equations represents a circle in the xy -plane that intersects the y -axis at exactly one point?

- A. $(x - 8)^2 + (y - 8)^2 = 16$
- B. $(x - 8)^2 + (y - 4)^2 = 16$
- C. $(x - 4)^2 + (y - 9)^2 = 16$
- D. $x^2 + (y - 9)^2 = 16$

ID: ff2ced2c Answer

Correct Answer: C

Rationale

Choice C is correct. The graph of the equation $x - h^2 + y - k^2 = r^2$ in the xy -plane is a circle with center h, k and a radius of length r . The radius of a circle is the distance from the center of the circle to any point on the circle. If a circle in the xy -plane intersects the y -axis at exactly one point, then the perpendicular distance from the center of the circle to this point on the y -axis must be equal to the length of the circle's radius. It follows that the x -coordinate of the circle's center must be equivalent to the length of the circle's radius. In other words, if the graph of $x - h^2 + y - k^2 = r^2$ is a circle that intersects the y -axis at exactly one point, then $r = h$ must be true. The equation in choice C is $x - 4^2 + y - 9^2 = 16$, or $x - 4^2 + y - 9^2 = 4^2$. This equation is in the form $x - h^2 + y - k^2 = r^2$, where $h = 4, k = 9$, and $r = 4$, and represents a circle in the xy -plane with center $4, 9$ and radius of length 4 . Substituting 4 for r and 4 for h in the equation $r = h$ yields $4 = 4$, or $4 = 4$, which is true. Therefore, the equation in choice C represents a circle in the xy -plane that intersects the y -axis at exactly one point.

Choice A is incorrect. This is the equation of a circle that does not intersect the y -axis at any point.

Choice B is incorrect. This is an equation of a circle that intersects the x -axis, not the y -axis, at exactly one point.

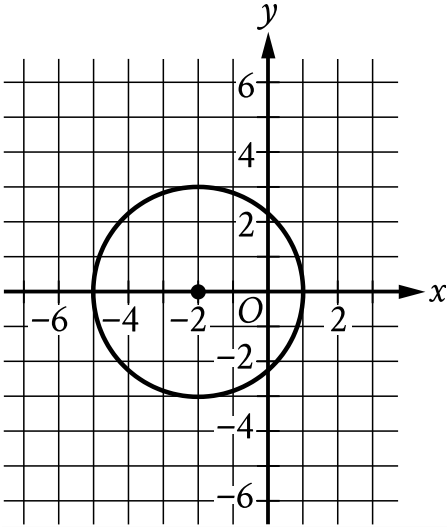
Choice D is incorrect. This is the equation of a circle with the center located on the y -axis and thus intersects the y -axis at exactly two points, not exactly one point.

Question Difficulty: Hard

Question ID 834ac03f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 834ac03f



Circle *A* (shown) is defined by the equation $(x + 2)^2 + y^2 = 9$. Circle *B* (not shown) is the result of shifting circle *A* down 6 units and increasing the radius so that the radius of circle *B* is 2 times the radius of circle *A*. Which equation defines circle *B*?

- A. $(x + 2)^2 + (y + 6)^2 = (4)(9)$
- B. $2(x + 2)^2 + 2(y + 6)^2 = 9$
- C. $(x + 2)^2 + (y - 6)^2 = (4)(9)$
- D. $2(x + 2)^2 + 2(y - 6)^2 = 9$

ID: 834ac03f Answer

Correct Answer: A

Rationale

Choice A is correct. According to the graph, the center of circle *A* has coordinates -2, 0, and the radius of circle *A* is 3. It's given that circle *B* is the result of shifting circle *A* down 6 units and increasing the radius so that the radius of circle *B* is 2 times the radius of circle *A*. It follows that the center of circle *B* is 6 units below the center of circle *A*. The point that's 6 units below -2, 0 has the same x-coordinate as -2, 0 and has a y-coordinate that is 6 less than the y-coordinate of -2, 0. Therefore, the coordinates of the center of circle *B* are -2, 0 - 6, or -2, - 6. Since the radius of circle *B* is 2 times the radius of circle *A*, the radius of circle *B* is 23. A circle in the *xy*-plane can be defined by an equation of the form $x - h^2 + y - k^2 = r^2$, where the coordinates of the center of the circle are *h*, *k* and the radius of the circle is *r*. Substituting -2 for *h*, -6 for *k*, and 23 for *r* in this equation yields $x - -2^2 + y - -6^2 = 23^2$, which is equivalent to $x + 2^2 + y + 6^2 = 2^23^2$, or $x + 2^2 + y + 6^2 = 49$. Therefore, the equation $x + 2^2 + y + 6^2 = 49$ defines circle *B*.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This equation defines a circle that's the result of shifting circle *A* up, not down, by 6 units and increasing the radius.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 3e05efb1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 3e05efb1

A circle has center G , and points M and N lie on the circle. Line segments MH and NH are tangent to the circle at points M and N , respectively. If the radius of the circle is 168 millimeters and the perimeter of quadrilateral $GMHN$ is 3,856 millimeters, what is the distance, in millimeters, between points G and H ?

- A. 168
- B. 1,752
- C. 1,760
- D. 1,768

ID: 3e05efb1 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the radius of the circle is 168 millimeters. Since points M and N both lie on the circle, segments GM and GN are both radii. Therefore, segments GM and GN each have length 168 millimeters. Two segments that are tangent to a circle and have a common exterior endpoint have equal length. Therefore, segment MH and segment NH have equal length. Let x represent the length of segment MH . Then x also represents the length of segment NH . It's given that the perimeter of quadrilateral $GMHN$ is 3,856 millimeters. Since the perimeter of a quadrilateral is equal to the sum of the lengths of the sides of the quadrilateral, $3,856 = 168 + 168 + x + x$, or $3,856 = 336 + 2x$. Subtracting 336 from both sides of this equation yields $3,520 = 2x$, and dividing both sides of this equation by 2 yields $1,760 = x$. Therefore, the length of segment MH is 1,760 millimeters. A line segment that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. Therefore, segment GM is perpendicular to segment MH . Since perpendicular segments form right angles, angle GMH is a right angle. Therefore, triangle GMH is a right triangle with legs of length 1,760 millimeters and 168 millimeters, and hypotenuse GH . By the Pythagorean theorem, if a right triangle has a hypotenuse with length c and legs with lengths a and b , then $a^2 + b^2 = c^2$. Substituting 1,760 for a and 168 for b in this equation yields $1,760^2 + 168^2 = c^2$, or $3,125,824 = c^2$. Taking the square root of both sides of this equation yields $\pm 1,768 = c$. Since c represents a length, which must be positive, the value of c is 1,768. Therefore, the length of segment GH is 1,768 millimeters, so the distance between points G and H is 1,768 millimeters.

Choice A is incorrect. This is the distance between points G and M and between points G and N , not the distance between points G and H .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the distance between points M and H and between points N and H , not the distance between points G and H .

Question ID db84ccdc

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: db84ccdc

A circle in the xy -plane has a diameter with endpoints $(2, 4)$ and $(2, 14)$. An equation of this circle is $(x - 2)^2 + (y - 9)^2 = r^2$, where r is a positive constant. What is the value of r ?

ID: db84ccdc Answer

Correct Answer: 5

Rationale

The correct answer is 5. The standard form of an equation of a circle in the xy -plane is $x - h^2 + y - k^2 = r^2$, where h , k , and r are constants, the coordinates of the center of the circle are h , k , and the length of the radius of the circle is r . It's given that an equation of the circle is $x - 2^2 + y - 9^2 = r^2$. Therefore, the center of this circle is 2, 9. It's given that the endpoints of a diameter of the circle are 2, 4 and 2, 14. The length of the radius is the distance from the center of the circle to an endpoint of a diameter of the circle, which can be found using the distance formula, $\sqrt{x_1 - x_2^2 + y_1 - y_2^2}$. Substituting the center of the circle 2, 9 and one endpoint of the diameter 2, 4 in this formula gives a distance of $\sqrt{2 - 2^2 + 9 - 4^2}$, or $\sqrt{0^2 + 5^2}$, which is equivalent to 5. Since the distance from the center of the circle to an endpoint of a diameter is 5, the value of r is 5.

Question Difficulty: Hard

Question ID 9ca6e7b4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9ca6e7b4

A circle has center O , and points A and B lie on the circle. The measure of arc AB is 45° and the length of arc AB is 3 inches. What is the circumference, in inches, of the circle?

- A. 3
- B. 6
- C. 9
- D. 24

ID: 9ca6e7b4 Answer

Correct Answer: D

Rationale

Choice D is correct. It’s given that the measure of arc AB is 45° and the length of arc AB is 3 inches. The arc measure of the full circle is 360° . If x represents the circumference, in inches, of the circle, it follows that $\frac{45^\circ}{360^\circ} = \frac{3 \text{ inches}}{x \text{ inches}}$. This equation is equivalent to $\frac{45}{360} = \frac{3}{x}$, or $\frac{1}{8} = \frac{3}{x}$. Multiplying both sides of this equation by $8x$ yields $1(x) = 3(8)$, or $x = 24$. Therefore, the circumference of the circle is 24 inches.

Choice A is incorrect. This is the length of arc AB .

Choice B is incorrect and may result from multiplying the length of arc AB by 2.

Choice C is incorrect and may result from squaring the length of arc AB .

Question Difficulty: Hard

Question ID 3a95868c

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 3a95868c

Point O is the center of a circle. The measure of arc RS on this circle is 100° . What is the measure, in degrees, of its associated angle ROS ?

ID: 3a95868c Answer

Correct Answer: 100

Rationale

The correct answer is 100. It's given that point O is the center of a circle and the measure of arc RS on the circle is 100° . It follows that points R and S lie on the circle. Therefore, $\overline{OR}$ and $\overline{OS}$ are radii of the circle. A central angle is an angle formed by two radii of a circle, with its vertex at the center of the circle. Therefore, $\angle ROS$ is a central angle. Because the degree measure of an arc is equal to the measure of its associated central angle, it follows that the measure, in degrees, of $\angle ROS$ is 100.

Question Difficulty: Hard

Question ID ddb62d63

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: ddb62d63

Point F lies on a unit circle in the xy -plane and has coordinates $(1, 0)$. Point G is the center of the circle and has coordinates $(0, 0)$. Point H also lies on the circle and has coordinates $(-1, y)$, where y is a constant. Which of the following could be the positive measure of angle FGH , in radians?

- A. $\frac{27\pi}{2}$
- B. $\frac{29\pi}{2}$
- C. 24π
- D. 25π

ID: ddb62d63 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the circle is a unit circle, which means the circle has a radius of 1. It's also given that point G is the center of the circle and has coordinates $(0, 0)$ and that point H lies on the circle and has coordinates $(-1, y)$. Since the radius of the circle is 1, the value of y must be 0, as all other points with an x -coordinate of -1 are a distance greater than 1 away from point G . Since F and H are points on the unit circle centered at G , let side FG be the starting side of the angle and side GH be the terminal side of the angle. Since side FG is on the positive x -axis and side GH is on the negative x -axis, side FG is half of a rotation around the unit circle, or π radians, away from side GH . Therefore, the positive measure of angle FGH , in radians, is equal to π plus an integer multiple of 2π . In other words, the positive measure of angle FGH , in radians, is an odd integer multiple of π . Of the given choices, only 25π is an odd integer multiple of π .

Choice A is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(0, -1)$, not $(-1, 0)$.

Choice B is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(0, 1)$, not $(-1, 0)$.

Choice C is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(1, 0)$, not $(-1, 0)$.

Question Difficulty: Hard

Question ID 55b41004

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 55b41004

A circle in the xy -plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k ?

- A. -3
- B. $-\frac{1}{3}$
- C. $\frac{1}{3}$
- D. 3

ID: 55b41004 Answer

Correct Answer: A

Rationale

Choice A is correct. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It's given that the circle has its center at $-4, -6$ and line k is tangent to the circle at the point $-7, -7$. The slope of a radius defined by the points q, r and s, t can be calculated as $\frac{t-r}{s-q}$. The points $-7, -7$ and $-4, -6$ define the radius of the circle at the point of tangency. Therefore, the slope of this radius can be calculated as $\frac{-6 - -7}{-4 - -7}$, or $\frac{1}{3}$. If a line and a radius are perpendicular, the slope of the line must be the negative reciprocal of the slope of the radius. The negative reciprocal of $\frac{1}{3}$ is -3 . Thus, the slope of line k is -3 .

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect. This is the slope of the radius of the circle at the point of tangency, not the slope of line k .

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 9bf2678d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Geometry and Trigonometry	Circles	Hard

ID: 9bf2678d

What is the value of $\sin 42\pi$?

- A. 0
- B. $\frac{1}{2}$
- C. $\frac{\sqrt{2}}{2}$
- D. 1

ID: 9bf2678d Answer

Correct Answer: A

Rationale

Choice A is correct. The sine of a number t is the y -coordinate of the point arrived at by traveling a distance of t units counterclockwise around the unit circle from the starting point $1, 0$. Since the unit circle has a circumference of 2π units, it follows that one full rotation around the circle is equal to a distance of 2π units. Therefore, a distance of 42π units around the circle from the starting point $1, 0$ would result in exactly 21 full rotations, arriving back at the point $1, 0$. So, $\sin 42\pi$ is equal to the y -coordinate of the point $1, 0$, which is 0.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the value of $\cos 42\pi$, not $\sin 42\pi$.

Question Difficulty: Hard